# Accretion Physics of AGN Disks

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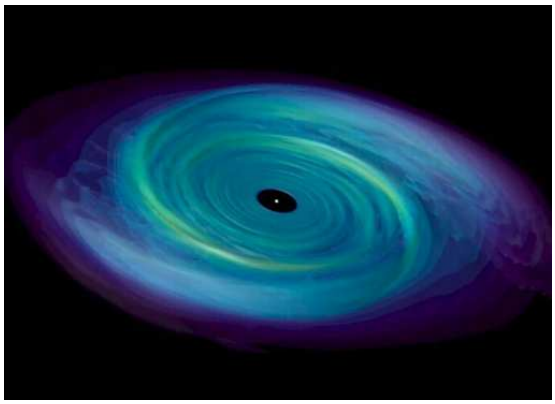


## General Outline

I. Disk Properties
- o Energy Considerations
- o Efficiency
- o Eddington Limit

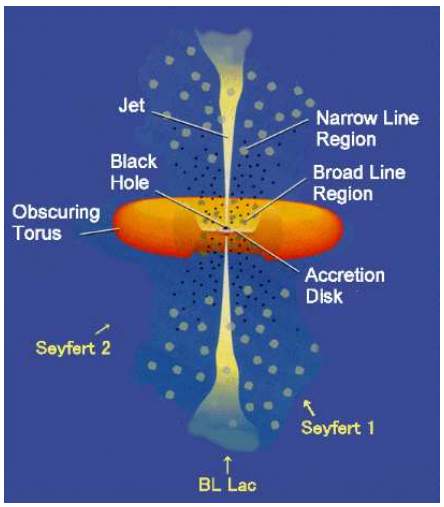
II. Thin Disk Accretion
- o Conditions for Formation
- o Viscosity and Torque
- o Temp./Emission Profiles

III. Fueling of Black Hole
- o Angular Momentum Transport

IV. Accretion Flows
- o Accretion Rate

**Scales**

BH ~ $10^{-5}$ pc
Disk (X–ray) ~ few x BH
Solar System ~ 0.0001 pc
Disk (UV) ~ 0.001 pc
BLR ~ 0.01 pc
Galaxy ~ $10^{4}$ pc
NLR ~ $10^{4}$–$10^{5}$ pc
Jets ~ $10^{5}$–$10^{6}$ pc




***o*** Disk Angular Scale ~ $10^{-6}$ arcsec.
***o*** HST resolution ~ 0.05 arcsec.

**=>** need ~ 50,000 times better resolution to image disk vicinity

## General Disk Properties

Equations

$$\frac{1}{2}m_{\rm p}v_{\rm ff}^2 = \frac{GMm_{\rm p}}{R}$$

$$L = \frac{1}{2}\dot{m}v_{\rm ff}^2 = \frac{GM\dot{m}}{R}$$

$$r_{\rm g} \equiv 2GM/c^2$$

$$L = \frac{1}{2}\dot{m}c^2\left(\frac{r_{\rm g}}{R}\right) = \xi\dot{m}c^2$$

$$\hookrightarrow \xi = \frac{1}{2}(r_{\rm g}/R)$$

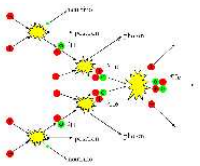
Efficiency Comparisons

$\xi_{\rm WD} \approx 3 \times 10^{-4}$

$\xi_{\rm NS} \approx 0.1$

$\xi_{\rm Schwartz} \approx 0.06$

$\xi_{\rm Kerr} \leq 0.426$

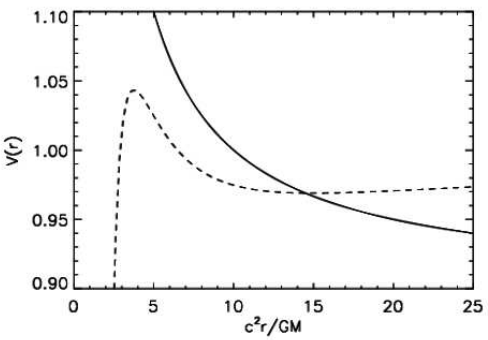


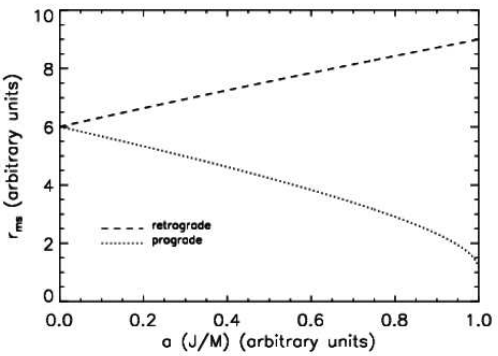
$\xi_{\rm pp\text{-}chain} \approx 7 \times 10^{-3}$

## Note On Stable Orbits

***o*** relativistic treatment of gravitational potential near Schwartzschild Black Hole shows last stable orbit is at $r=3r_g$.

***o*** inside this orbit material will quickly plunge into the black hole. $r < 3r_g$ is defined as the 'plunging region.'




***o*** a spinning black hole (Kerr black hole) can shift the last stable orbit inward or outward dependent on how the angular momentum vectors of the black hole and material align.

## Eddington Limit

$$f_{\rm grav} \approx \frac{GMm_{\rm p}}{r^2}$$

$$f_{\rm rad} = \frac{\sigma_{\rm T} L}{4\pi r^2 c}$$

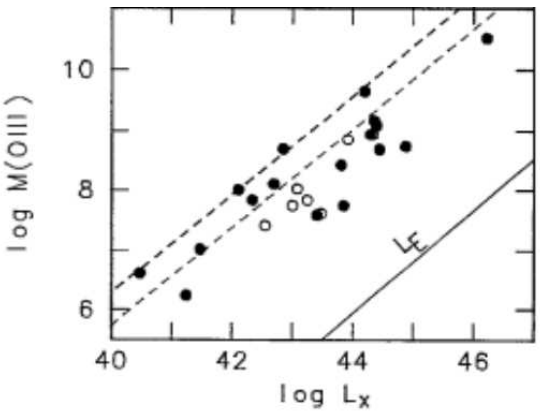
$$f_{\rm grav} = f_{\rm rad}$$

$$\hookrightarrow L_{\rm Edd} = \frac{4\pi GMm_{\rm p}c}{\sigma_{\rm T}}$$

$$= 10^{38}\left(\frac{M}{M_\odot}\right) \text{ erg s}^{-1}$$

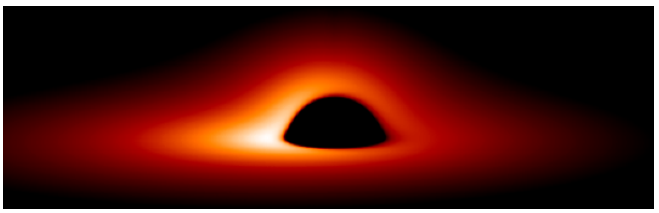
$$\hookrightarrow M \leq \frac{L_{\rm obs}\sigma_{\rm T}}{4\pi G m_{\rm p} c}$$

– AGN with dynamical mass estimates show most of these types of black holes radiate well below the Eddington limit.




<–– Eddington–based mass constraint.

## Thin Disk Accretion



### Formation Conditions

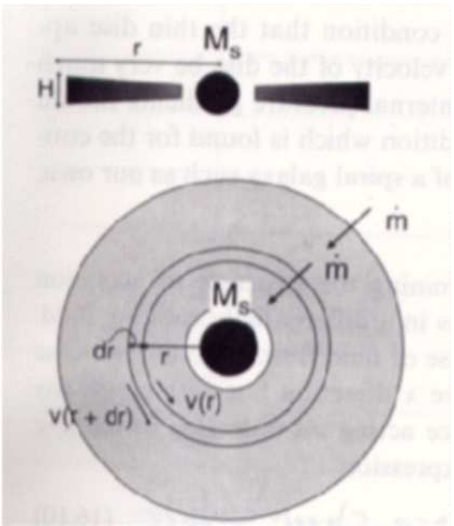
$$\frac{\partial p}{\partial z} = -\frac{GM_c \rho sin\theta}{r^2}$$

$$\frac{p}{H} = \frac{GM_c \rho H}{r^3}$$

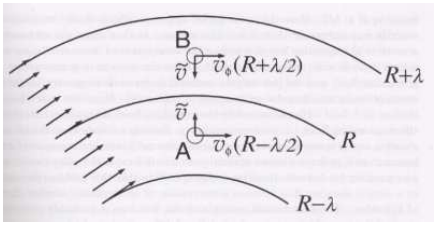
$$v_\phi^2 = \frac{GM}{r} \text{ and } c_{\rm s}^2 = p/\rho$$

$$\Rightarrow \frac{H}{r} \approx \frac{c_{\rm s}}{v_\phi}$$

$$\Rightarrow v_\phi \ll c_{\rm s}$$



## Viscosity and Torque ;)




local approximation ...

$\lambda \equiv$ turbulence length scale

$\tilde{v} \equiv$ characteristic turbulent velocity

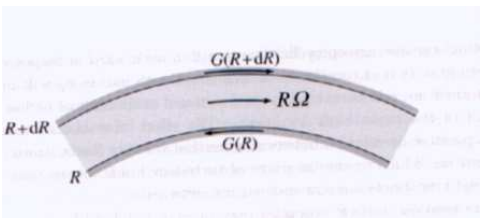
$$\sigma_{r\phi} = -\eta R\frac{\partial\Omega}{\partial R} \sim -\rho\tilde{v}\lambda R\frac{\partial\Omega}{\partial R}$$

$$G(R) = \sigma_{r\phi} \times \text{Area} \times R$$

$$\text{Area} \approx 2\pi RH$$

$$\Sigma \equiv H\rho \text{ and } \nu \equiv \eta/\rho \sim \tilde{v}\lambda$$

$$\Rightarrow G(R) = -2\pi R\Sigma\nu R^2\frac{\partial\Omega}{\partial R}$$




for a ring at $R$ with a width $dR$

$$G_{\text{net}} = G(R+dR) - G(R) = \frac{\partial G}{\partial R}dR$$

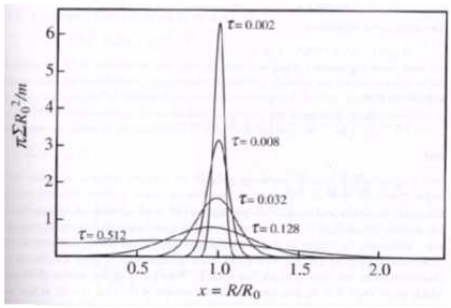
$$\Omega\frac{\partial G}{\partial R}dR = \frac{\partial}{\partial R}(G\Omega)dR - G\frac{\partial\Omega}{\partial R}dR$$

$$D(R) = G\frac{\partial\Omega}{\partial R}dR \times 1/\text{Area}$$

$$\text{Keplerian Disk: } \Omega_K(R) = \left(\frac{GM}{R^3}\right)^{1/2}$$

$$\Rightarrow D(R) = \frac{9}{8}\nu\Sigma\frac{GM}{R^3}$$

## Radial Disk Structure




$$M_{\text{annulus}} = 2\pi RdR\Sigma$$

$$\ell_{\text{annulus}} = 2\pi RdR\Sigma R^2\Omega$$

Conservation of Mass:

$$R\frac{\partial\Sigma}{\partial t} + \frac{\partial}{\partial R}(R\Sigma v_R) = 0$$

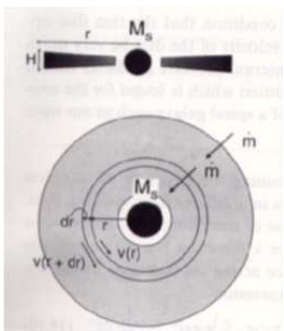
Conservation of Angular Momentum:

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega) + \frac{\partial}{\partial R}(R\Sigma v_R R^2\Omega) = \frac{1}{2\pi}\frac{\partial G}{\partial R}$$

$$\Rightarrow R\Sigma v_R\frac{\partial(R^2\Omega)}{\partial R} = \frac{1}{2\pi}\frac{\partial G}{\partial R} \text{ for } \frac{\partial\Omega}{\partial t} = 0$$

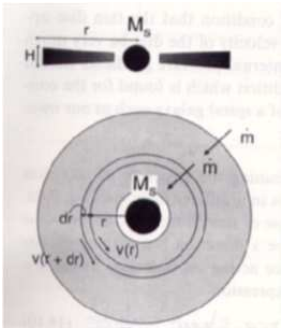
$$R\frac{\partial\Sigma}{\partial t} = -\frac{\partial}{\partial R}(R\Sigma v_R) = -\frac{\partial}{\partial R}\left[\frac{1}{2\pi}\left(\frac{\partial(R^2\Omega)}{\partial R}\right)^{-1}\frac{\partial G}{\partial R}\right]$$

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu\Sigma R^{1/2})\right]$$



## Steady Thin Disk Temperature Profile

$$\frac{\partial}{\partial t} = 0$$




$$R\Sigma v_R = constant \ldots \text{Mass Conservation}$$

$$\dot{M} = -2\pi R\Sigma v_R$$

$$R\Sigma v_R R^2\Omega = \frac{G}{2\pi} + \frac{C}{2\pi} \ldots \text{Momentum Conservation}$$

$$C = 2\pi R_*^3 \Sigma v_R \Omega(R_*) \text{ at inner boundary}$$

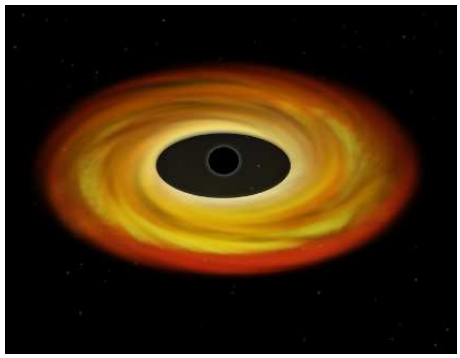
$$\text{or } C = -\dot{M}(GMR_*)^{1/2}$$

$$\Rightarrow \nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

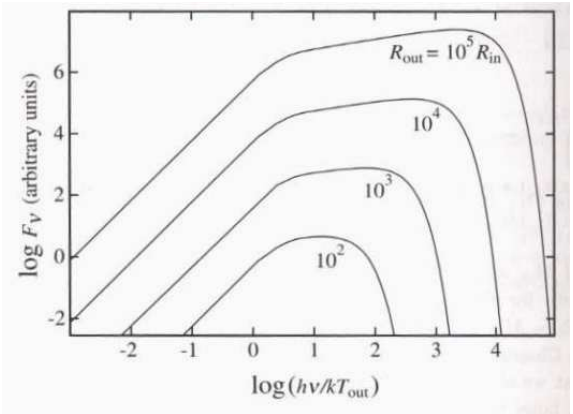
$$\text{recall } \ldots \ D(R) = \frac{9}{8}\nu\Sigma\frac{GM}{R^3}$$

$$\sigma T^4 = D(R) \text{ Optically Thick}$$

$$\Rightarrow T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3\sigma}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4}$$



## Disk Emission




$$I_\nu = B_\nu[T(R)] = \frac{2h\nu^3}{c^2(e^{h\nu/kT} - 1)}$$

$$F_\nu = \frac{2\pi\cos i}{D^2}\int_{R_*}^{R_{out}} I_\nu R dR$$

$$\nu \ll kT(R_{out})/h \ldots$$

$$F_\nu \propto \nu^2$$

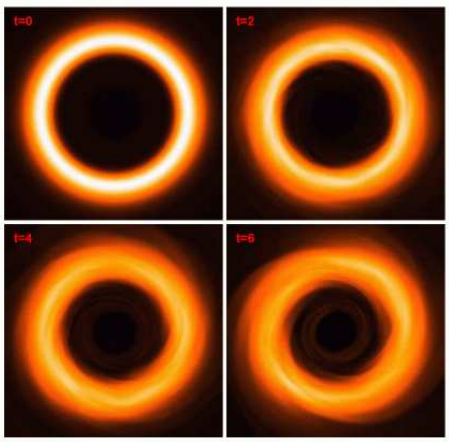
$$\nu \gg kT(R_*)/h \ldots$$

$$F_\nu \propto \nu^3 e^{-h\nu/kT}$$

$$kT(R_{out})/h \ll \nu \ll kT(R_*)/h \ldots$$

$$F_\nu \propto \nu^{1/3}$$

## Angular Momentum Transport




***o*** In order for the outer disk to accrete it must lose ~ 99% of its angular momentum.

***o*** Pure hydrodynamical disks satisfy Rayleigh criterion:

$$\frac{\mathrm{d}}{\mathrm{d}R}(R^2\Omega) > 0$$

***o*** Weak disk magnetic fields could provide reasonable viscosity to drive accretion.

**Magnetorotational Instabilities**

***o*** For weak vertical magnetic fields the disk becomes unstable when:

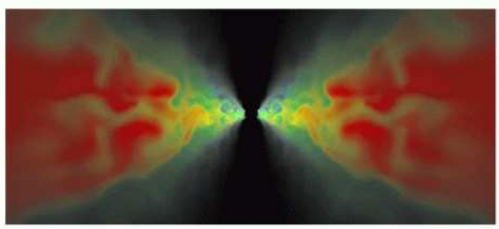
$$\frac{\mathrm{d}\Omega^2}{\mathrm{d}\,\ln R} < 0$$

## Effect of Accretion Rate

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\xi c^2} \text{ (maximal accretion rate)}$$

$$\dot{m} \equiv \dot{M}/\dot{M}_{\rm Edd}$$

$$\dot{m} \ll 1 \ ...$$



***o*** radiative losses will go into heating the ions and the two will equillibrate due to Coulomb interactions on timescales ...

$$\tau_{ei} = \frac{3(kT_e)^{3/2} m_p}{8ne^4\sqrt{2\pi m_e}\,\ln\Lambda}$$

***o*** For low accretion rate it is thought $t_{ei} >> t_{\rm accretions}$ and a two–temperature plasma may exist and the following disk properties are expected...

- geometrically thick flow
- efficiency will be lower than thin–disk case
- some of the material may outflow from the nucleus