Astronomy 485 — Problem Set 5 — Weeks 10 and 11

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All problems are worth 10 points.

1. (a) A pulsar is observed to have a gamma-ray luminosity of L watts and a pulse period of P seconds. Estimate a lower limit on $\dot{P}$. If both P and $\dot{P}$ are measured discuss how a limit on the distance to the pulsar can be obtained from its gamma-ray flux (i.e., calculate an inequality that the distance must satisfy). You may assume isotropic gamma-ray emission, although this may not be the case in reality.

1. (b) Estimate the scale height (the distance over which the density increases by a factor of e) of the (iron) atmosphere of a 1.4 $M_\odot$ neutron star with surface temperature 10^6 K. You may assume the atmosphere is an ideal gas.

1. (c) Suppose a spherical body spinning originally at a rate ω_1 with a radius R_1 collapses gravitationally to a radius R_2, conserving its mass M and angular momentum J. Express the ratios of the new and old spin rates ω_2/ω_1 and the new and old rotational energies E_2/E_1 in terms of the ratio R_1/R_2 (you may assume the moment of inertia for a homogeneous sphere). By what factor would the core of a star spin faster if it were to collapse from a radius typical of a white dwarf to the dimensions typical of a neutron star? By what factor would the rotational energy increase in such a collapse? Where ultimately does this energy come from?

2. (a) A neutron star cannot spin with less than a certain critical period or it will start to shed mass from its equator due to centrifugal force. Consider a neutron star of mass M and radius R. Show that it will shed mass if its period is less than

$$P_{\rm min} = K \left(\frac{1.4\ M_\odot}{M}\right)^{1/2} \left(\frac{R}{10\ \rm km}\right)^{3/2}\ \rm ms$$

where K is a constant. You may assume Newtonian gravity, and you may neglect deformation of the neutron star due to its rotation and magnetic field. What is the numerical value of K?

2. (b) A more detailed calculation that includes General Relativity and other effects gives $K = 0.77$ (see *Nature*, 340, 617 if you are interested). Look at the World Wide Web site `http://www.atnf.csiro.au/research/pulsar/psrcat/` to find the name and period of the pulsar with the shortest spin period. Using the equation above with $K = 0.77$ and $M = 1.4\ M_\odot$, calculate the limit on the radius of this neutron star. This limit is important because it constrains the equation of state for neutron star matter.

3. (a) The eclipsing binary X-ray source SMC X-1 lies in the Small Magellanic Cloud. It is an X-ray pulsar. We detect ≈ 50 X-rays per second from it in a detector which has an aperture of 400 cm^2. The X-rays have a typical energy of 5 keV. Estimate the X-ray luminosity of SMC X-1.

3. (b) The orbital period of SMC X-1 is 3.892 days, and the maximum pulse time delay is ± 53.46 seconds. Optical spectroscopy suggests its companion to have a mass of $\approx 17 M_\odot$ and gives a radial velocity amplitude of 19 km s^{-1}. Estimate the mass of the neutron star (please do not just assume an inclination angle of 90 degrees, since not all eclipsing systems are inclined to this degree). Note that this neutron star seems to be radiating either anisotropically or at a super-Eddington rate.

4. (a) Consider an LMXB that shows X-ray bursts. Estimate the interval between bursts if the quiescent accretion luminosity is 1% of $L_{\rm Edd}$. Assume a neutron star mass of 1.4 $M_\odot$ and a burst α parameter (see page 21 of the Fabian notes) of 70. You may model the burst as a 5-second 'spike' at $L_{\rm Edd}$.
4. (b) Use the Stefan-Boltzmann law to estimate the maximum blackbody temperature (in keV) reached during a burst.
4. (c) Estimate the maximum distance (in kpc) at which an old (single) neutron star can be detected by *Chandra* if it is moving through the interstellar medium at 20 km s^{-1} and has a mass of 1.4 M$_\odot$. Take the number density of the interstellar medium to be 1 cm^{-3}. The *Chandra* sensitivity level is about 4×10^{-16} erg cm^{-2} s^{-1} in a 10^5 s exposure. You may assume that the neutron star emits all of its flux in the *Chandra* band.

5. Please write an ≈ 2 page report on magnetars. Describe some recent discoveries about magnetars as well as the earlier work on these objects. I have put a relevant *Scientific American* article on the course World Wide Web page, and I'd suggest looking at `http://solomon.as.utexas.edu/magnetar.html` and `https://arxiv.org/abs/1703.00068`. Many other World Wide Web sites can be found using one of the standard search engines. Your report should address the following issues as well as other ones you think are important.

- What is the basic theoretical picture for magnetars? How might their extreme magnetic fields be created?
- What is the typical magnetic field of a magnetar, and how does this compare to the strongest magnetic fields that people can make?
- Why do magnetars spin so slowly?
- What important discovery was made on March 5, 1979, and how is it related to magnetars?
- What satellites and other facilities contributed to the discovery of the connection between soft gamma-ray repeaters and magnetars? How many soft gamma-ray repeaters have now shown pulsations?
- How do magnetars make soft gamma-ray outbursts? Do they follow the Gutenberg-Richter Law and, if so, why is this interesting?
- What happened on August 27, 1998 and on December 27, 2004?
- How many magnetars are currently known? How many more might there be in our Galaxy?

Please document your work by giving the references that you used.

6. The precise equation for gravitational wave radiation from a circular binary system is

$$L_{\rm GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}$$

where μ is the reduced mass and M is the total mass of the binary system (compare with page 26 of the Fabian notes).

Consider two dead pulsars (each of mass 1.4 $M_\odot$) in a circular orbit with separation a. Over time, the binary orbit will shrink due to gravitational wave radiation, and eventually the two dead pulsars will crash together (likely to make one of the types of classical gamma-ray bursts—a totally different phenomenon from the soft gamma-ray repeaters you read about for Problem 5). Show that the time t_0 until $a \to 0$ is

$$t_0 = \frac{5}{256}\frac{c^5}{G^3}\frac{a_{\rm now}^4}{M^2\mu}.$$

For the orbital dynamics you may assume that Newtonian gravity is valid until $a \to 0$ (although in reality the situation is somewhat more complicated). Then show that, to order of magnitude, t_0 is given by

$$t_0 \approx 10^5 \left(\frac{P_{\rm now}}{1\ \rm second}\right)^{8/3} \quad \text{seconds}$$

where $P_{\rm now}$ is the observed orbital period. How long does a ten-minute binary system consisting of two dead pulsars last? If you are stuck you may want to look at pages 476–477 of *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects* by S.L. Shapiro and S.A. Teukolsky.