Astronomy 485 — Problem Set 3 — Weeks 5 and 6

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All problems are worth 10 points.

1. (a) Detector dead time. If the count rate from a bright X-ray source is high enough, the X-ray detector may miss a legitimate count because it is still processing a previous count. This is known as detector dead time; the detector is 'dead' (insensitive to further counts) for a time Δt after registering a count. Work out a simple equation for the actual count rate R' in terms of the measured count rate R and Δt. Counts that are missed do not themselves cause dead time. If we measure a source to have a count rate of $R = 100$ count s^{-1} and the dead time is $\Delta t = 1$ ms, what is the actual count rate?

1. (b) The *ASCA* satellite weighed 420 kg. It was launched in 1993 by an ISAS M-3S-II rocket into a low Earth orbit with an altitude of about 550 km. Using Kepler's laws, find the approximate orbital period that *ASCA* had (in minutes).

2. Ionized hydrogen accreted onto a white dwarf passes through a shock close to the surface at radius $R_\star$. Show that the shock temperature is approximately given by

$$\frac{kT}{m_{\rm e}c^2} = \frac{3}{32}\frac{m_{\rm p}}{m_{\rm e}}\frac{R_{\rm S}}{R_\star}.$$

Here $R_{\rm S} = \frac{2GM}{c^2}$ is the Schwarzschild radius. As in class, you may treat the pre-shocked matter as if it were 'cold.' What is kT (in keV) if $R_\star = 6000$ km and $M = 1M_\odot$?

3. We have learned about the Eddington limit as the characteristic luminosity when gas is supported by radiation pressure against the gravitational attraction of a collapsed mass M. Here we will calculate a few other characteristic 'Eddington quantities.' Consider a black hole of radius $r_{\rm g} = \frac{GM}{c^2}$. Consider Eddington limited accretion of ionized hydrogen onto this black hole with unit efficiency (for the conversion of mass into radiant energy). Hence we have $\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{c^2}$.

3. (a) The Eddington number density. Derive the characteristic number density $n_{\rm Edd}$ when matter is flowing radially through the black hole event horizon at nearly the speed of light (adopt a spherical accretion geometry). Show that

$$n_{\rm Edd} \approx 1 \times 10^{19} \left(\frac{M}{M_\odot}\right)^{-1}$$

where $n_{\rm Edd}$ is measured in units of cm^{-3}.

3. (b) The Eddington time. Calculate the numerical value (in years) of the quantity $M/\dot{M}_{\rm Edd}$. What is the physical meaning of this quantity (i.e., by what factor does the black hole mass increase in this time)?

4. (a) The Cygnus superbubble is about 15° in diameter and is at a distance of ≈ 2 kpc. If its temperature is $\approx 2 \times 10^6$ K and its luminosity is 5×10^{36} erg s^{-1}, please estimate the energy content of the gas. You may assume that the superbubble is spherical and homogeneous. You may also assume solar abundances and neglect line emission. Could the energy content of the gas have come from just the photons emitted by one supernova?

4. (b) What is the cyclotron frequency of an electron in a starspot with a magnetic field of 4000 gauss? In what band of the spectrum is this frequency?

4. (c) The star Algol (β Per) is at a distance of 27 pc. An X-ray flare on Algol is observed to have a temperature of 6×10^7 K and a flux of 8×10^{-11} erg cm^{-2} s^{-1} at the Earth. If the radiative decay of the emission takes 25,000 s, what is the electron density, volume and radius of the emission region? You may assume a hemispherical geometry for the flare and a uniform density within this volume. You may also assume solar abundances and that bremsstrahlung dominates the flare emission.

5. Recall the fine structure constant (α) and the gravitational fine structure constant ($\alpha_{\rm G}$). We have seen how many important physical quantities contain these two constants.

5. (a) Show that the Rydberg constant can be written as $\frac{mc^2}{2}\alpha^2$. You should use cgs units for this problem.

5. (b) Show that the Bohr radius, the Compton wavelength, and the classical radius of the electron scale approximately as $1{:}\alpha{:}\alpha^2$. Do not worry about factors of 2π that may emerge as a result of h versus $\hbar$.

5. (c) In class we have used the Bohr radius several times. Here we will derive the Bohr radius in a simple way that will be useful later. Apply the uncertainty principle to a hydrogen atom where the electron and proton are separated by distance a. Show that its radius is $a_0 \approx \hbar^2/m_{\rm e}e^2$ by minimizing the total energy (as a function of a). Do not worry if your answer is off by a numerical factor of less than ~ 10. This problem does not require any highly technical quantum mechanics calculations.

Note: The point of this problem is to derive the Bohr radius approximately in a way different from that in most introductory physics books. Please do *not* follow the standard path presented in many introductory physics books but instead use the method above.

6. Please read the article ‘Confirmation of Earth-Mass Planets Orbiting the Millisecond Pulsar PSR B1257+12’ by A. Wolszczan (available from the course World Wide Web page in Portable Document Format). Although we have not yet covered pulsars in detail, this article should be clear, and it discusses some interesting data analysis methods. Then please write an ≈ 2 page essay describing the main results of this article. You should address issues such as the following:

- What are the basic observational and analysis techniques that were used to find the planets? How was chisquared fitting used in this work?
- How does this paper present very strong evidence that the planet interpretation is correct?
- How does the velocity resolution of pulsar timing compare to that of optical Doppler spectroscopy?
- How might these planets have formed?
- How might we find more such planets?