Astronomy 485 — Problem Set 1 — Weeks 1 and 2

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All problems are worth 10 points.

1. (a) Please approximately calculate the hydrogen column density through your chest. You may assume that you are made of water.
1. (b) Given your result to part (a) and the discussion of photoelectric absorption in class, do you think a doctor could use 100 eV X-rays to make pictures of your bones? Why or why not?
1. (c) What is the speed of an electron with a kinetic energy of 300 keV? What is the de Broglie wavelength of this electron?

2. Consider an iron atom that has been stripped of all but one of its electrons.
2. (a) Using the Bohr atomic model (or a more sophisticated one if you prefer), please calculate the energy (in keV) of the line photon created when the electron falls from its $n = 2$ atomic level to its $n = 1$ atomic level. This is the famous iron Kα line that is frequently used in X-ray astronomy, and it is worth remembering this energy.
2. (b) Now please calculate the threshold energy of the absorption edge created when the electron is photoionized from the $n = 1$ level. This is the famous iron K edge.
2. (c) If the atom had 2 electrons instead of just one, would the edge threshold energy be higher or lower (no need to calculate the exact number here, just reason it out using physics)? Please explain your answer.
2. (d) The iron Kα line is one of the strongest emission lines studied by X-ray astronomers. One of the reasons for its strength is the relatively high abundance of iron. Based on your reading of the Garmire notes pages 1–8, state another reason for the strength of this line. Please fully explain the reasoning behind your answer. There are actually 2 possible answers, and either one will do.

3. (a) Please graph the 0.05–50 keV emission spectrum of a blackbody with temperature $kT = 2$ keV on a plot of energy (in keV) versus flux density (in keV cm^{-2} s^{-1} keV^{-1}). Assume the emitting object is a sphere of radius 10 km at a distance of 0.2 kpc. At approximately what energy (in keV) does the flux density from this blackbody peak?

Please make your plot reasonably precise (i.e. *not* just a rough sketch); use of a computer is allowed and encouraged. Please use logarithmic scales for the x and y axes of your plot.

For this problem, you may neglect the photoelectric absorption of X-rays in the interstellar medium.

Also, if you are having trouble going from flux density at the surface of the sphere to detected flux density, check out section 1.3 of the book by Rybicki & Lightman.

3. (b) Please repeat the above for a blackbody with temperature $kT = 100$ keV. However, now make your plot for the 1–2000 keV range.
3. (c) Do either of these sources emit many photons above the threshold for electron-positron pair production (roughly twice the mass of the electron in keV)?

4. (a) Circinus X-1 is a bright X-ray source in our Galaxy. It is a binary system where a neutron star is in orbit around a normal star. By analyzing the X-ray spectrum of Circinus X-1, we find a hydrogen column density of 2×10^{22} cm^{-2}. If the average density of the gas toward Circinus X-1 is 3×10^{-24} gm cm^{-3}, please estimate the distance (in kpc) to Circinus X-1.
4. (b) If we instead measured a column of 2×10^{24} cm^{-2} would you believe your distance result? Why or why not?
4. (c) Find the distance (in kpc) in the Galaxy for an attenuation of $1/e$ at energies of 0.25, 0.50, 1.0, 2.0 and 5.0 keV if the mean density in the plane of the Galaxy is 0.5 atoms of hydrogen per cm^3 (i.e. the flux has dropped to $1/e$ times its initial value). Use the analytic cross section for photoelectric absorption we covered in class. Compare each of these distances to the distance to the center of our Galaxy (about 8.5 kpc).

5. (a) On page 3 of the Garmire notes, the energy radiated by a thermal plasma per unit volume and unit time is calculated. Note that the lower limit for this integral is not 0, like you might expect, but is rather $\sqrt{\frac{2h\nu}{m_e}}$. By thinking about the physics of how the photons are being created, explain the reason for this nonzero lower limit using a simple argument.
5. (b) Please perform the $j(\nu, T)$ integral and reduce the resulting equation to get the simplified equation on page 3 of the Garmire notes (i.e. the one starting with 6.8×10^{-38}). Note that you can just pull the Gaunt factor out of the integral since it is velocity averaged already.

6. A large solar flare can be roughly modeled as a cylindrical region some 5×10^4 km in diameter by about 5×10^4 km high at a temperature of 10^7 K. The flux from such a region is about $\frac{1}{10}$ erg cm^{-2} s^{-1} at the Earth integrated over all energies. What is the electron density, energy density, and cooling timescale for such a plasma? If the magnetic field in the flaring region is 2×10^3 gauss, how does the magnetic energy density compare to the energy density in the plasma?

7. (a) Consider a spherical cloud of ionized hydrogen near a supermassive black hole with $n_e = n_i = 1 \times 10^8$ cm^{-3}. Take the temperature and radius of this cloud to be 2 keV and 1×10^{11} cm, respectively, and assume that it radiates only via the bremsstrahlung process. Using the equation for $j(\nu, T)$ discussed in class (and in the Garmire notes), please draw a graph of the luminosity density of this cloud (in keV s^{-1} keV^{-1}) versus energy (in keV).[1] Please make your plot reasonably precise (i.e. *not* just a rough sketch). For the Gaunt factor, please use the $(kT/h\nu)^{0.4}$ approximation discussed on page 3 of the Garmire notes.
7. (b) Please repeat the above for a cloud temperature of 50 keV.
7. (c) Please calculate the total luminosity (in erg s^{-1}) of the cloud discussed in part (a). Now, if the same total mass of ionized hydrogen is spread among 4 equal mass spherical clumps, each with radius 1×10^{10} cm, what is the total luminosity? Has it gone up or down (or remained the same) relative to the unclumped case?

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[1] 'Luminosity density' is just the luminosity equivalent of flux density—nothing tricky is meant by this term. This quantity is also sometimes referred to as 'monochromatic luminosity.'

8. For all questions below, please clearly explain your answer giving your reasoning when appropriate.
8. (a) H-like ions have strong Lyman-α (i.e. $n = 2$ to $n = 1$) transitions at energies of $\approx 10.2Z^2$ eV. However, this simple scaling is not very accurate for high Z ions. For example, the Kα line energy of H-like iron is actually at 6.97 keV rather than 6.90 keV. State one reason why the simple scaling becomes inaccurate.
8. (b) What is an Einstein A coefficient? What is the order of magnitude of this coefficient for a typical X-ray line?
8. (c) What is a satellite line and how is it made? How is dielectronic recombination relevant to the production of such lines?
8. (d) What is collisional autoionization?
8. (e) Why is dielectronic recombination suppressed at high densities?
8. (f) What are the physical conditions that define a coronal plasma? Are coronal plasmas in true thermodynamic equilibrium?
8. (g) What is resonance scattering?
8. (h) What is the characteristic timescale for energy exchange between electrons by elastic Coulomb scattering?
8. (i) Give two examples of photoionized plasmas found in nature.
8. (j) What is the ionization parameter of an optically-thin photoionized plasma? Why is the ionization parameter a useful quantity?

Hint: The Canizares notes should be helpful for these questions.

9. Please read the article 'The *Chandra* X-ray Observatory (CXO)' by M.C. Weisskopf (astro-ph/9912097). You should get this article from the World Wide Web sites http://arxiv.org/abs/astro-ph/9912097 (the text in PDF or Postscript format) and http://arxiv.org/ps/astro-ph/9912097 (the figures in color JPEG format). Then please write an $\approx$ 2 page essay describing some of the key points of this article. You should address issues such as the following:

- What are some of the basic, key science goals that drove the construction of *Chandra*? How has its design allowed it to achieve these science goals?
- What are the main scientific instruments on *Chandra* and roughly how do they work?
- How is *Chandra* superior to *XMM-Newton*, and how is it inferior to *XMM-Newton*? How do these two satellites complement each other?
- What were some of the ground tests performed to ensure that *Chandra* would work properly when launched?
- Who is able to use *Chandra*? Where is the *Chandra* data archive?

Don't worry if you do not understand some of the technical details of this article. I am asking you to read it this early to immerse you in some of the recent exciting events in X-ray astronomy. The technical details will be clear by the end of the course. Also feel free to check out and use the World Wide Web sites listed at the end of this article for helpful pictures and explanations.

You might want to explore the astro-ph World Wide Web site at arxiv.org more generally too; you'll be using this a lot if you become an astronomer.

10. How many X-ray photons (say, 0.1–100 keV) will you emit in your lifetime? Please be quantitative, even if the number is small.