

Emission Processes¹ for High-Energy Astrophysics.

by Gordon Garmire

The Concept of a Cross Section and Related Terms.

In order to discuss how light or particle beams are attenuated in passing through a material, it is convenient to introduce the concept of an interaction cross section. In the simplest sense, the cross section is just the projected area of a particle or atom of the material. For example, if the material is modeled by a collection of hard spheres of radius R , then the cross section of each sphere for scattering light rays, imagined to be little beams of light, is just πR^2 . This cross-section is often referred to as the total scattering cross-section. The light ray could either be completely absorbed by the sphere, partially scattered into some new direction, or reflected completely by some scattering mechanism such as specular reflection.

Absorption Length

Consider a flux of radiation of intensity I_0 particles/area/time impinging upon a container of atoms each with cross sectional area πR^2 . If the atoms have a density n per unit volume and are randomly distributed in the volume then in a depth dx , there will be ndx atoms per unit area (ignore atoms that are cut into pieces by the ends of dx). The fraction of a unit area covered by atoms will be $n\pi R^2 dx$. This area will be shadowed by the atoms if we assume that they are all perfect absorbers. The intensity after passing through dx will be less by an amount

$$\delta I = -I_0 n \pi R^2 dx.$$

To find the effect of a number of such thin absorbing layers, we can sum up the intensity decreases by integrating with respect to x . This gives

$$\int_{I_0}^{I(x)} \frac{dI}{I} = -n\pi R^2 \int_0^x dx$$

If we now define

$$\ell = 1/(n\pi R^2)$$

we can write

$$I(x) = I_0 \exp(-x/\ell)$$

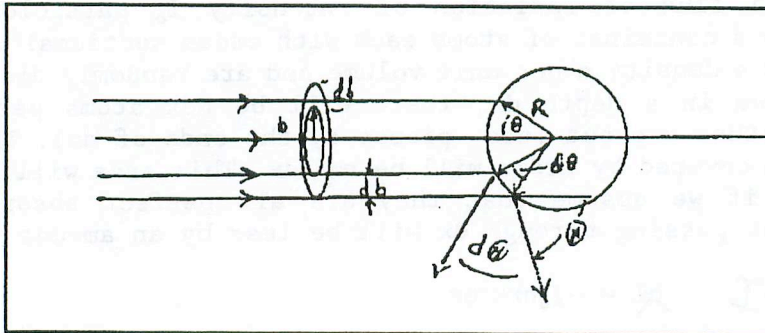
where ℓ becomes the absorption length. We have ignored the shadowing of one atom by another in going from one dx element to the next. Since dx is arbitrarily small, this is justified so long as n is not infinitely large. A more sophisticated approach would use the probability of absorption in dx and treat each layer in a probabilistic manner. Such a complication is not needed at this point in our discussion.

The absorption length is sometimes referred to as optical depth for

absorption, i.e., if we go into the material a distance ℓ , we are at a unit optical depth for absorption. If the atoms or absorbers are not perfect, but can scatter the radiation as well as absorb it and there is enough scattering to place a substantial amount of radiation back into the original beam, then the problem is much more complicated and we must develop a method of radiative transfer. For most of the applications in this course, such a complication will not be required.

Differential Scattering Cross Section

In the above discussion we assumed that the atoms acted as perfect absorbers and that the total projected area of each atom was all that was important for the absorption process. In many applications it is necessary to consider a more complicated situation, namely, that the radiation may be scattered into a different direction without being absorbed (a mixture of absorption and scattering is also possible, of course). A simple way to treat the scattering problem for our example of a reflecting sphere (something like a shiny ball bearing, for example) is to examine the following diagram. Suppose a beam of radiation is incident upon our sphere as shown.



Let the beam be uniform in intensity such that the flux in little ring is simply $I2\pi b db$, where $2\pi b db$ is the area of the ring, centered about the ray passing through the center of the sphere. It is straight forward to derive a relationship between the distance b (sometimes called the "impact parameter") and the angles θ and θ , namely $b=R \sin \theta$ and $\theta=180-2\theta$. We can relate the area $2\pi r b db$ to R and θ from geometry as

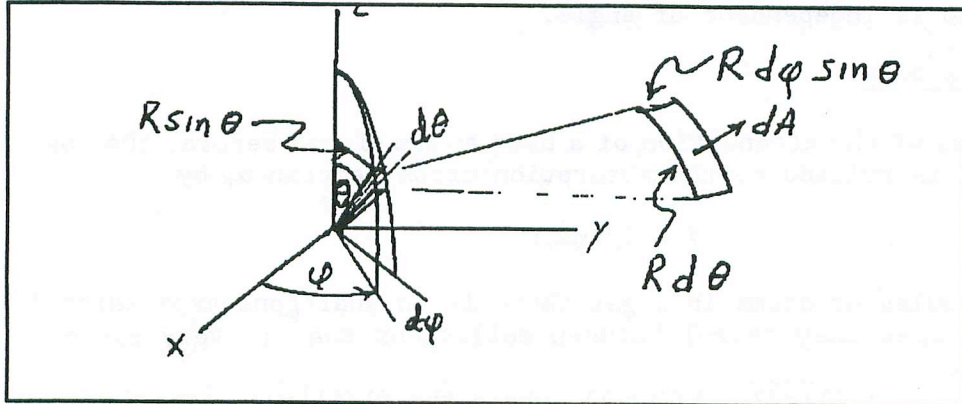
$$2\pi b db = 2\pi R \sin \theta \cos \theta d\theta$$

for $b \leq R$.

At this point it is useful to introduce a new concept (to some of you, perhaps), that of the solid angle.

As you can see from the above figure above, for a given b and db , rays are scattered into a range of angles between θ and $\theta+d\theta$, all around the axis of symmetry through the center of the sphere. The angle around the symmetry axis is usually called ϕ . The unit of solid angle is

called the steradian. There are 4π steradians over the entire sphere. One way of thinking about the steradian is to consider the area of a sphere of unit radius. The area of such a sphere is then 4π area units. A differential element of area on such a sphere is given in the diagram below.



$$dA = (R \sin \theta d\phi) R d\theta$$

Now for $R = 1$ we have

$$d\Omega = \sin \theta d\theta d\phi$$

and

$$\int_{\text{sphere}} d\Omega = 2 \int_0^{\pi/2} \int_0^{2\pi} \sin \theta d\theta d\phi = 4\pi \text{ steradians}$$

We now can introduce the concept of a differential cross section, $\sigma(\Omega)$ or $d\sigma/d\Omega$ as it is sometimes written in the literature, defined as:

$$\sigma(\Omega) d\Omega = \frac{\text{particles scattered into } d\Omega \text{ per unit time}}{\text{incident flux per unit area}}$$

For the case of our reflecting sphere we have

$$2\pi b db = 2\pi R^2 \sin \theta \cos \theta d\theta$$

Using $\theta = \pi - 2\theta$ this becomes

$$2 \cos \theta \sin \theta d\theta = -\cos(\theta/2) \sin(\theta/2) d\theta$$

or using 1/2 angle formulas

$$\sigma(\Omega) d\Omega = (1/2)\pi R^2 \sin \theta d\theta$$

(and noting that $d\Omega = 2\pi \sin \theta d\theta$ for cylindrical symmetry)

$$\sigma(\Omega) = (1/4) R^2 d\Omega$$

This expression shows that $\sigma(\Omega)$ is independent of angle, i.e. the shiny ball scatters uniformly. The total cross section is obtained by integrating this expression over all angles, which gives 4π , since the integrand is independent of angle.

Mean Free Path

In terms of the attenuation of a beam as mentioned before, the absorption length l is related to the absorption cross section σ_T by

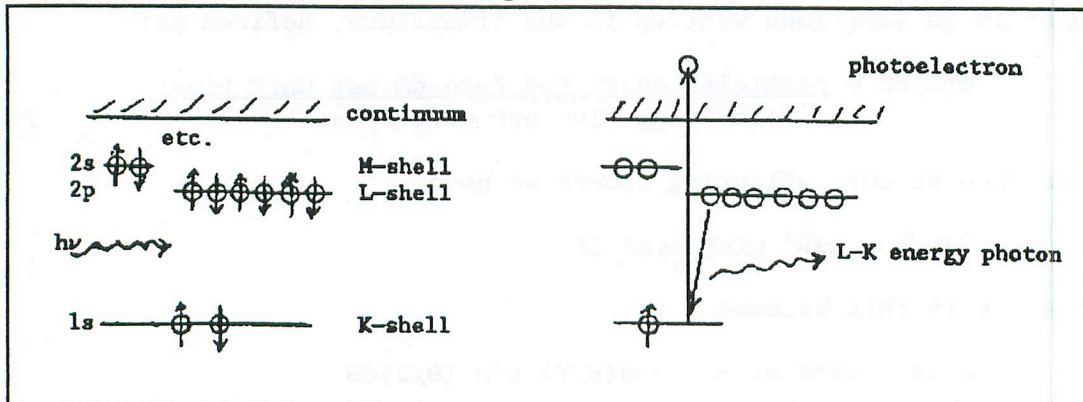
$$l = 1/(n\sigma_T)$$

For molecules or atoms in a gas there is an analogous expression for the mean distance they travel between collisions that is very close to this expression,

i.e., $l_{mfp} = (1/4)\sqrt{\pi/2} (1/(n\sigma_T))$, where the $(1/4)\sqrt{\pi/2}$ comes from averages taken over Maxwellian velocity distributions for the atoms which are moving, and the fact that the beam atoms have a cross sectional area of σ_T as well as the target atoms.

Photoelectron Absorption Cross section

In the x-ray band of the spectrum, atoms absorb x-rays primarily by photoelectric absorption of the x-ray photon with the subsequent emission of an electron from the atom leaving an ion and an excited atom, which ultimately de-excites with the emission of a photon of energy characteristic of the atomic energy levels. A simple energy diagram of an atom might look like the figure below



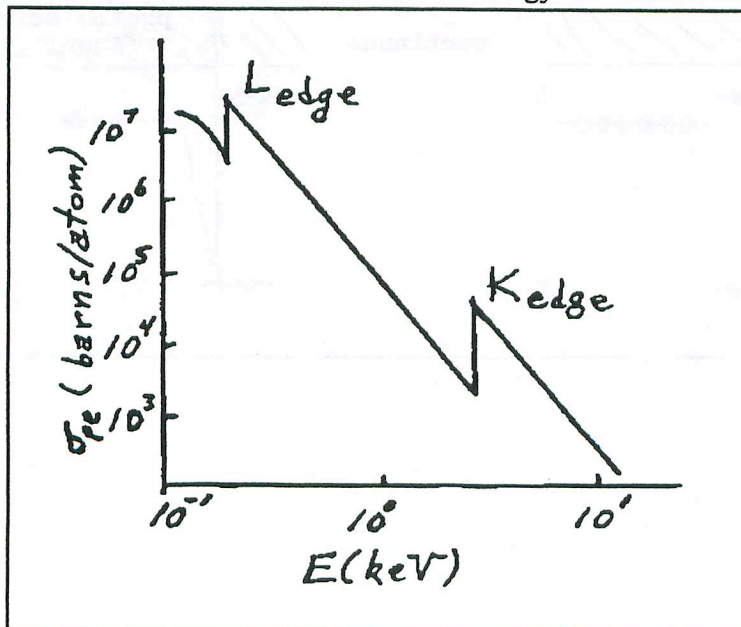
The levels in an atom are referred to as shells in x-ray literature and are denoted as above. The famous Einstein equation applies to this interaction, namely

$$h\nu - \phi = E_{ke \text{ max}}$$

where $h\nu$ is the incoming photon energy, ϕ is the binding energy of the electron in the atom which is ejected and $E_{ke \text{ max}}$ is the maximum kinetic energy of the emitted electron. The most probable electron to be ejected is the one deepest in the atom's potential well consistent with energy conservation. The kinetic energy of the emitted electron is exactly determined in the case of an isolated atom. The photo-electric equation given above is more general in that it allows the electron to be scattered before leaving a substance, in which case it could lose some energy before reaching a detector. The L-K energy photon emitted when $h\nu$ is greater than the K shell binding energy, is the most likely photon emitted. Higher energy photons will occasionally be emitted (M-K energy) as well as further filling of holes until the ion reaches its ground state energy.

The calculation of the cross section for this process is a standard quantum mechanics problem (see L. Schiff, Quantum Mechanics). Essentially the atom is treated by perturbation theory. Usually a hydrogenic ion is assumed, so that the wave functions for the atom can be expressed in a simple form. We will only use the results of the calculation at this time, as well as measured data. A good source for absorption cross sections is W. J. Veigeli - 1973 Atomic Data Vol. 5 p52, or Henke et al. 1982 Atomic Data Tables 27, 1.

In general, the photo-electric cross section is strongly energy dependent and depends on the atomic charge of the atom. For hydrogenic atoms, $\sigma_{pe} \sim Z^5/E^{7/2}$. For neutral atoms σ_{pe} goes more like Z^4 and $E^{-8/3}$. A typical cross section as a function of energy is shown below.

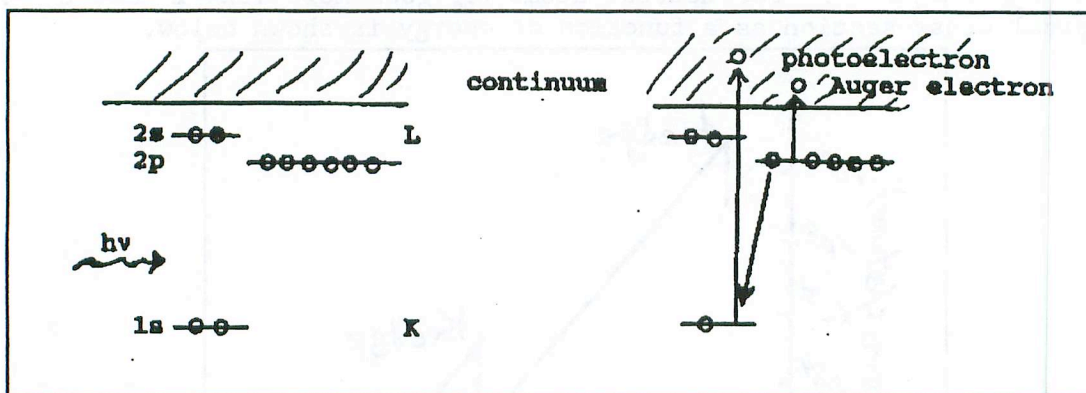


The K-edge is produced when the photon energy falls below the binding energy of the K-shell electrons in the atom. At this energy only L-shell electrons can be ejected, and since the photon energy is much larger than the L shell energies the interaction is not as strong as it was just at the K-shell energies where a resonance occurs. The cross section is usually given in units of barns, which are units of 10^{-24} cm². This is a small unit for atoms and a large unit for nuclear collisions. If a gas is made up of several kinds of atoms, each one acts independently of the others for absorbing x-rays. The total absorption is obtained by simply adding up each species with its absorbing cross section and density.

The subsequent de-excitation of the atom deserves some comment. There are actually two ways that the de-excitation takes place: a) by the emission of a photon or b) by the emission of an Auger (pronounced O-jay) electron. The relative probability of these two effects depends upon the Z of the atom and the energy levels involved. Case a) is sometimes called the fluorescent yield of an atom. For L to K-shell transitions the fraction of atoms de-exciting by the emission of a photon is empirically given as

$$C_k = Z^4 / (Z^4 + 10^6) .$$

This shows that for high Z, mostly fluorescent photons are emitted (photons characteristic of the material being excited) and for low Z mostly Auger electrons are emitted. The atomic diagram for Auger electron emission looks like the following ;



There can be a whole cascade of Auger electrons as the atom adjusts back to its ground state energy of the final ionic species.

Absorption by Interstellar Gas.

Most of the gas in the galaxy is atomic hydrogen. Because the photoelectric cross section increases so rapidly with increasing atomic charge, however, the less abundant elements such as helium, carbon and oxygen play a significant role in absorbing x-rays as they travel through the gas. Several workers have computed the absorbing power of the interstellar material. These are:

- Brown and Gould 1970 Phvs. Rev. DI, 2252
- Cruddace et al 1974 Ap.J. 187, 497
- Ride and Walker 1977 A. & A. 61, 339
- Morrison and McCammon 1983 Ap. J 270, 119.

For rough calculations an approximate analytic expression for the cross section is

$$\sigma_{pe} = 2 \times 10^{-22} (E/1\text{keV})^{-8/3} \text{ cm}^2 / \text{atom of hydrogen.}$$

This does not take account of absorption edges, however, only the edge at 532 eV for oxygen is significant compared to most detector energy resolution properties. For sources that have a spectral energy distribution that can safely be extrapolated to lower energies, the absorption of the x-rays together with an estimate of the mean density of hydrogen along the line of sight to the source can be used to estimate the distance to the source. Conversely, if the distance is known by some other means, the absorption provides information about the amount of matter between the earth and the source. Binary x-ray sources may have a substantial amount of gas in the vicinity of the stars, which can lead to a high estimate for the distance or density of matter in the interstellar medium.

The absorption for the interstellar medium is shown below from the Cruddace et al. reference.

Non Relativistic Bremsstrahlung and Thermal Bremsstrahlung

A very hot plasma with a temperature of over a million degrees is largely ionized. The radiation from such a plasma is composed of three components; radiation from the electrons when they collide with ions in the plasma, radiation from free electrons when they are captured by an ion, and radiation from ions that are not completely stripped of their electrons when they collide with free electrons. In a typical high temperature plasma that is in equilibrium, the electrons are moving much faster than the ions since they are much lighter but share the ions energy through the equipartition of energy, i.e. $\frac{1}{2}kT$ per degree of freedom where k is Boltzmann's constant and T the temperature. To obtain a rough estimate of the radiated power in a collision between an ion of charge Z and an electron with impact parameter b , the projected distance of closest approach of the electron to the ion, use the maximum acceleration times an effective collision time, i.e.,

$$a_{\max} = Ze^2/m_e b^2 \text{ and } \tau_{\text{coll.}} \approx b/v .$$

The radiation emitted will then be

$$\frac{dU}{dt} = \frac{2}{3} \frac{e^2}{c^3} a^2$$

$$U(b) \approx \overline{\frac{dU}{dt}} \tau_{\text{coll}}$$

$$\frac{dU}{dt} = \frac{2}{3} \frac{e^2}{c^3} \left(\frac{Ze^2}{m_e b^2} \right)^2$$

where use has been made of the general radiation formula for a non relativistic electron undergoing acceleration a .

For most of the situations of interest to x-ray astronomy we will be interested in how much power emerges from a unit volume of hot plasma, rather than the radiative loss of electron energy in a single collision. Let the plasma be composed of ions of number density N_z and electrons of number density $N_e > N_z$. In thermal equilibrium the electrons will move much faster than the ions, so we only need to consider their motion. They will have a Maxwellian velocity distribution $f(v)$ in thermal equilibrium. For the moment only consider electrons of speed v selected from the distribution. The power radiated by these electrons will be

$$\frac{dP(v)}{dV} = N_e N_z V \int_{b_{\min}}^{b_{\max}} U(b) 2\pi b db$$

The value for b_{\max} is much greater than b_{\min} in general, which means

$$\approx \frac{4\pi}{3} Z^2 e^2 r_e^2 N_e N_z C \left(\frac{1}{b_{\min}} - \frac{1}{b_{\max}} \right).$$

the upper limit can be set to infinity. The rule is that b should be of order of the Debye length in the plasma $\sim (kT/4\pi N_e e^2)^{1/2}$. This is the distance in a plasma that is required for the charges in a plasma to adjust their distribution to cancel out the effects of a localized charge i.e. the plasma is polarizable somewhat like a dielectric material. The value of b_{\min} is the size of the uncertainty wave packet for the electron characteristic of its motion, i.e., $b_{\min} \sim h/m_e v$. The velocities need to be greater than about $Z\alpha c$, or the electron will be captured by the ion.

Using this value for b_{\min}

$$\frac{dP(v)}{dv} = \frac{4\pi}{3} Z^2 r_e^2 e^2 C N_e N_z \frac{m_e v}{h}, \quad r_e = \frac{e^2}{m_e c^2}$$

The effective cross section for this process is defined as

$$\sigma = \left(\frac{dP(v)}{dv} \right) / N_z N_e v E, \quad E = \frac{1}{2} m_e v^2$$

That is to say, the cross section is the power radiated by particles of energy E and velocity v into all directions, divided by the flux of energy incident ($N_e v E$) on targets of density N_z . Writing this out,

$$\sigma = \frac{8\pi}{3} Z^2 r_e^2 e^2 C / \hbar m_e v^2$$

and using $\alpha = e^2 / \hbar c$, $\hbar = h / 2\pi$,

$$\sigma = \frac{8\pi}{3} Z^2 r_e^2 \alpha (c/v)^2$$

and substituting $\lambda_e = \hbar / m_e v$

$$\sigma = \frac{4}{3} Z^2 \alpha^3 \lambda_e^2$$

where the value of $\lambda_e \sim b_{\min}$.

An exact calculation using quantum mechanics gives

$$\sigma = \frac{16\pi}{3\sqrt{3}} Z^2 \alpha^3 \lambda_e^2 g_B(v, v)$$

where $g_B(v, v)$ is called the Gaunt factor after the man who first calculated this correction factor.

The Gaunt factor is given by

$$g_B(v, \nu) = \frac{\sqrt{3}}{\pi} \left(\frac{v_i}{v_f} \right) \frac{1 - e^{(-2\pi\eta_i)}}{1 - e^{-(2\pi\eta_f)}} \log \left(\frac{v_i + v_f}{v_i - v_f} \right), \quad h\nu = \frac{1}{2} m_e (v_f^2 - v_i^2), \quad v_i = v$$

where v_i and v_f are the initial and final velocities of the electron, and η_i and η_f are $Ze^2/\hbar v_i$ and $Ze^2/\hbar v_f$ respectively.

In order to compute the power radiated by a unit volume of plasma with a Maxwellian distribution of velocities, we must define the velocity distribution.

$$N(v) dv = 4\pi N_e \left(\frac{m_e}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{m_e v^2}{2kT}} v^2 dv$$

The energy radiated by the plasma per unit volume and time is then

$$j(\nu, T) = N_Z \int_{\sqrt{\frac{2h\nu}{m_e}}}^{\infty} N(v_i) h\nu \sigma(v_i, \nu) v_i dv_i$$

$$= \frac{64\pi}{3\sqrt{3}} N_e N_Z Z^2 \left(\frac{e^2}{c} \right)^2 r_e^2 c \left(\frac{c}{v_{th}} \right) \overline{g_{ff}}(\nu, T) e^{-\frac{h\nu}{kT}}$$

$$= 6.8 \times 10^{-38} Z^2 N_e N_Z T^{-\frac{1}{2}} \overline{g_{ff}}(\nu, T) e^{-\frac{h\nu}{kT}} \text{ ergs/cm}^3 \text{ secHz}$$

where $v_{th} = (2kT/\pi m_e)^{\frac{1}{2}}$ and $\overline{g_{ff}}(\nu, T)$ is the temperature averaged Gaunt factor. This factor has been computed by Karzas and Latter, 1961, Ap.J. Suppl. 6, 167. For the region of temperature and frequencies that we will study $\overline{g_{ff}}(\nu, T) \sim (kT/h\nu)^{0.4}$ to sufficient accuracy.

In the interstellar medium, the gas is composed of a small fraction of higher Z elements compared to hydrogen. If the plasma in an object is assumed to be composed of "cosmic abundance" of elements then we can compute the emission $j(\nu)$ for the sum of all these elements. Since the emission goes as $N_e N_Z$, and almost all N_Z will be completely ionized at the higher temperatures of x-ray sources, we will get Z electrons from a species N_Z . Letting

$$S = \sum_Z Z^2 N_e N_Z$$

$$= 1.4 N_e^2 \quad \text{for } T \geq 10^5 \text{ }^\circ\text{K.}$$

Total Emission

The total emission from a region emitting by thermal bremsstrahlung is just the integral of $j(\nu)$ over frequency. This integration including the Gaunt factor is

$$j(T) = \frac{32}{3\sqrt{3}} \alpha r_e^2 c \left(\frac{c}{v_{th}} \right) kT g_{ff}(T) N_e N_z Z^2$$

$$= 1.4 \times 10^{-27} T^{3/2} S g_{ff}(T) \text{ ergs/cm}^3 \text{ sec.}$$

The Gaunt factor is 1.2 for temperatures greater than 10^6 °K. Using S for cosmic abundance

$$j(T) = 2.4 \times 10^{-27} T^{3/2} N_e^2 \text{ ergs/cm}^3 \text{ sec.}$$

This emission dominates line emission and recombination radiation for temperatures above 10^7 °K. The energy content of the plasma is

$$E \approx 3 N_e kT \text{ (electron plus ions)}$$

so that the time scale for cooling is

$$\begin{aligned} \tau_{\text{eff}} &= 3 N_e kT / j(T) \\ &= 1.8 \times 10^{11} T^{3/2} / N_e \text{ sec.} \end{aligned}$$

Free-free Absorption

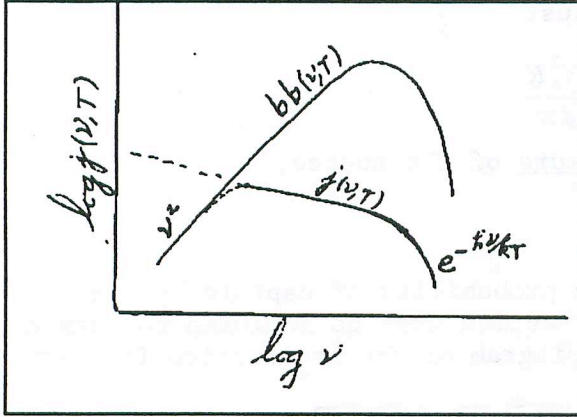
If there is a sufficient flux of photons in a plasma, it is possible for an electron to absorb a photon as it passes a nucleus rather than emitting a photon. This process is called free-free absorption, and the electron gains energy in the collision. The calculation of this process is carried out most easily by developing the equations of radiative transfer and requiring that the emission from a region does not exceed the emission from a blackbody at the same temperature and frequency. The results of such a computation is an expression for the free-free absorption.

$$\begin{aligned} \mu(\nu) &= \left(\frac{c^2}{2h\nu^3} \right) \left(e^{\frac{h\nu}{kT}} - 1 \right) (j(\nu, T) / 4\pi) \\ &= 3.7 \times 10^8 \nu^{-3} \left(1 - e^{-\frac{h\nu}{kT}} \right) T^{-1/2} N_e N_z g_{ff}(\nu, T) / (\nu^2 T^{-3/2}) \text{ cm}^{-1} \end{aligned}$$

In most x-ray sources this is important only for frequencies $h\nu \ll kT$ so that

$$\mu(\nu) = 0.018 Z^2 N_e N_z g_{ff}(\nu, T) \nu^{-2} T^{-3/2} \text{ cm}^{-1}$$

The effect of this factor is to modify the free-free emission spectrum as shown below



If a cloud of plasma is of radius R , then where $\mu(\nu) R$ becomes of order 1, the effect of free-free absorption will be to modify the spectrum to force it to equal or fall below the blackbody spectrum of the cloud if it were optically thick and emitting like a blackbody.

Emission Integral and Emission Measure

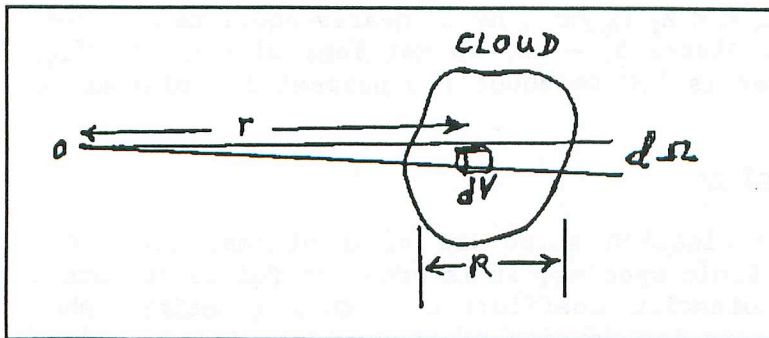
For media transparent to their own radiation field, the total emission from a hot cloud of gas will be proportional to the integral of N_e over the cloud.
i.e.

$$\langle n_e^2 V \rangle = \int_{\text{cloud}} N_e^2 dV$$

which is called the emission integral.

The surface brightness of a cloud of gas that can be resolved, or a cloud that surrounds the earth will be proportional to

$$\left(\frac{dI}{d\Omega} \right) d\Omega \propto \int_{\text{cloud}} \frac{N_e^2 r^2 dr d\Omega}{4\pi r^2}$$



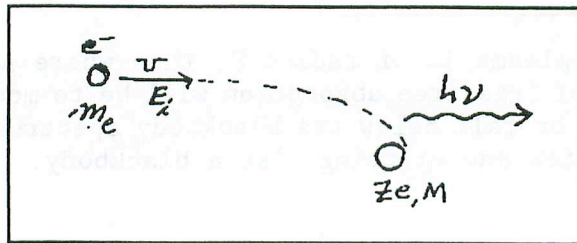
since from a point 0 the emission is proportional to the elements of volume and the intensity reaching 0 will be the emitted flux divided by $4\pi r^2$ if it is emitted isotropically. If we assume a uniform density in the cloud then the surface brightness is just

$$\frac{dI}{d\Omega} \propto \frac{N_e^2 R}{4\pi}$$

The quantity $N_e^2 R$ is the emission measure of the source.

Radiative Recombination

For electrons moving with $v < Z\alpha c$, the probability of capture by the ion is significant and this capture cross section must be included to obtain the emission from the hot plasma. The diagram of the interaction is shown below



Energy conservation provides

$$h\nu = E_i - I_{Z-1, n}$$

where $I_{Z-1, n}$ is the ionization energy of the ion with charge $Z-1$, after capture, and the principal quantum number n , is for the captured electron. The potential well that the electron falls into for hydrogenic ions has a depth $\Delta E_n = Z^2 I_H / n^2$, where I_H is the ionization energy of hydrogen to the ground state ($n = 1$, $I_H = 13.6$ eV).

The cross section is quite similar to the bremsstrahlung cross section, except for a term which takes account of the final state of the electron - i.e., it is bound.

$$\sigma_R(n) = \frac{32\pi}{3\sqrt{3}} Z^2 \alpha^3 \lambda_e^2 \left(\frac{Z^2 I_H}{h\nu} \right) g_R(n) / n^3$$

For low electron energies $E_i \ll Z^2 I_H / n^2$, $h\nu$ is nearly equal to $Z^2 I_H / n^2$, so $\sigma_R(n) \sim n^{-1}$. In a thermal plasma $E_i = kT$, so for ions with $kT \ll Z^2 I_H$, $\sigma_R(n) \sim n^{-1}$. The Gaunt factor is 1.0 to about ten percent for plasmas we will study.

Total Recombination Coefficient

In determining the ionization structure of a plasma, i.e., the concentration of different ionic species, it is often useful to compute a quantity called the recombination coefficient. This quantity, when multiplied by the electron and ion density of the plasma will provide a measure of the rate at which recombination is taking place.

The total recombination rate is

$$\alpha_r = \sum_n \langle \sigma_R(n) v \rangle$$

where the average is over the velocity distribution of the plasma

electrons. Carrying out this sum and averaging yields

$$\alpha_r = \frac{64\pi}{3\sqrt{3}} \alpha^3 a_0^2 \left(\frac{2kT}{\pi m_e} \right)^{1/2} y \phi(y) \bar{g}$$

$$= 2 \times 10^{-11} Z^2 T^{-1/2} \phi(y) \text{ cm}^3 \text{ sec}^{-1}$$

where $y = Z^2 I_H/kT$ and a_0 is the Bohr radius, $\hbar^2/m_e e^2$.

$$\begin{aligned} \phi(y) &= 0.5 (1.735 + \ln y + \frac{1}{6y}), \quad y \geq 1 \\ &= y(-1.202 \ln y - 0.298), \quad y < 1 \end{aligned}$$

At temperatures for which $kT \gg Z^2 I_H$

$$\alpha_r \rightarrow Z^4 T^{-3/2} \ln(kT/Z^2 I_H),$$

showing that recombination falls rapidly when the electron energy becomes large compared to the binding energy of the ion.

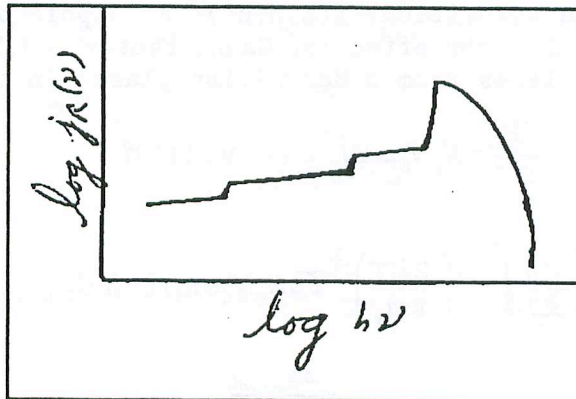
Continuum Spectrum from Recombination

The spectrum of recombination radiation is given by

$$j_R(\nu) = N_Z h \nu \sigma_R(n) \int_0^\infty N_e(\nu) \nu d\nu$$

$$= 1.8 \times 10^{-32} T^{-3/2} n^{-3} N_e N_Z Z^4 \exp((I_{Z-1,n} - h\nu)/kT) \text{ ergs/cm}^3 \text{ sec Hz}$$

which will look like



Line Radiation

Line radiation occurs following the excitation of an atom or ion either by an electron collision or the absorption of a photon of the line energy. Since the latter process does not yield new photons to the radiation field unless a cascade process follows the excitation, the collision process will be evaluated here to determine the emission of energy from the plasma.

In the case of line radiation, an electron must transfer a precise energy to the atomic electron. As a function of impact parameter this energy can be evaluated using an impulse approximation as we have done for free-free emission.

$$\Delta P_{\perp} \sim F \Delta \tau$$

$$\sim \frac{e^2}{b^2} \frac{2b}{v}$$

$$\Delta E \sim \frac{\Delta P_{\perp}^2}{2m_e} \sim \frac{e^4}{b^2 v^2 m_e}$$

which corresponds to the line energy. This give a value for b of

$$b^2 \sim \frac{2e^4}{m_e v^2 \Delta E} \sim \frac{e^4}{E_e \Delta E}$$

where ΔE will be related to the line energy. The cross section is then of order (less than actually, since only a narrow range of b's will transmit the correct energy)

$$\sigma(\Delta E) \approx \pi b^2.$$

Now if we introduce the Bohr radius

$a_0 = \hbar^2/m_e e^2$ and the ionization potential of hydrogen $I_H = e^2/2a_0$, we can write the cross section as

$$\sigma(\Delta E) = 4 \pi a_0^2 I_H^2 / (E_e \Delta E).$$

The correct quantum mechanical calculation gives

$$\sigma(\Delta E_{z,nn'}) = \frac{8\pi^2}{\sqrt{3}} \frac{a_0^2 I_H^2}{E \Delta E_{z,nn'}} \bar{F}(Z, n, n') \bar{g}(Z, n, n', E_z)$$

for a hydrogenic atom of charge Z, where n and n' are the principal quantum numbers of the transition, $f(Z, n, n')$ the dipole oscillator strength and $g(Z, n, n', E_z)$, the effective Gaunt factor (-0.2 for hydrogen).

The emission of lines from a Maxwellian plasma is then

$$\frac{dP_{\perp}}{dV} = N_e N_z \Delta E \int_0^{\infty} \sigma(v) v f(v) dv$$

$$= \frac{32\pi^2}{\sqrt{3}} a_0^2 \left(\frac{I_H}{kT} \right)^2 kT \left(\frac{2kT}{\pi m_e} \right)^{\frac{3}{2}} \bar{F} \bar{g} N_e N_z \exp(-\Delta E_{z,nn'}/kT)$$

$$= 2.3 \times 10^{-15} T^{-\frac{1}{2}} \bar{F} \bar{g} N_e N_z e^{-\frac{\Delta E_{z,nn'}}{kT}} \text{ ergs/cm}^3 \text{ sec.}$$

If we sum over all lines for a plasma of cosmic abundance composition, then the total radiative power and separate components of the power for

the three emission processes are shown below

For a plasma with a cosmic abundance of elements, the plasma emission is shown for three temperatures in the following diagram. These figures are from Giacconi and Gursky, X-ray Astronomy.

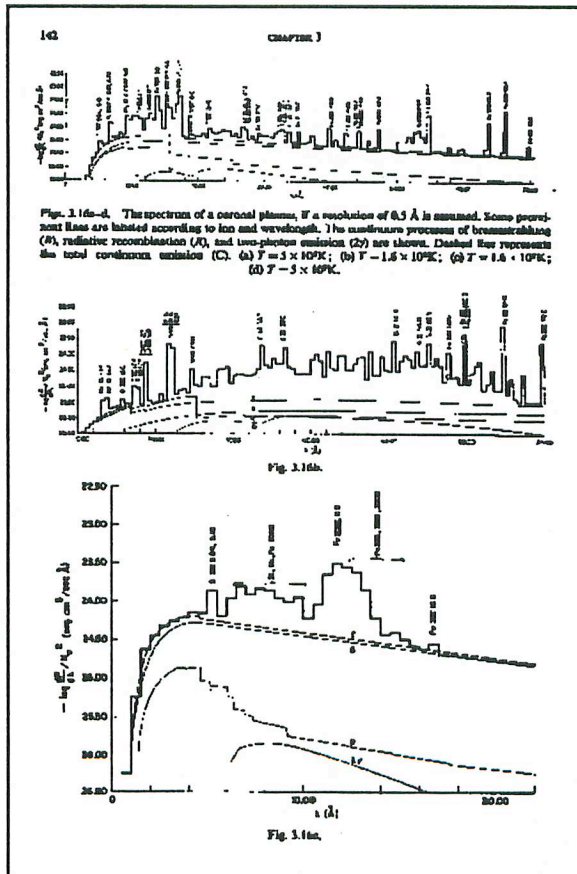
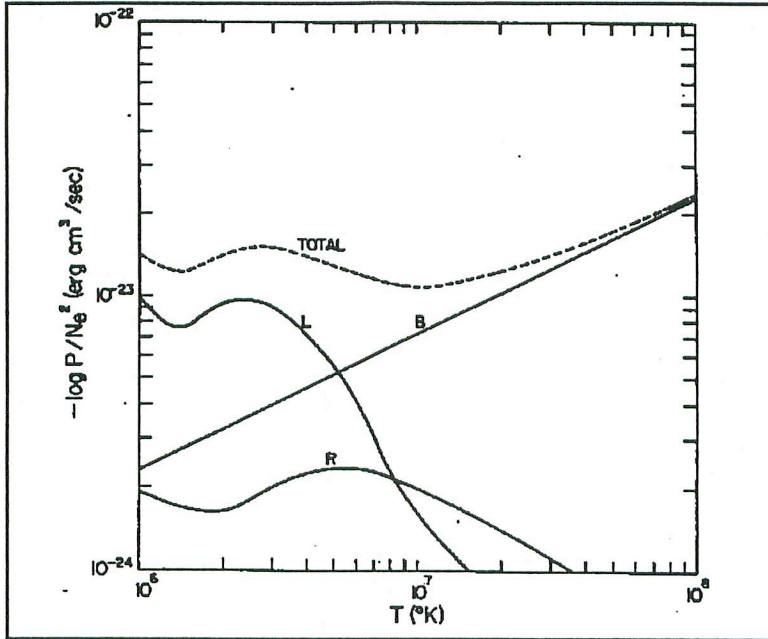


Fig. 3.16a-d. The spectrum of a coronal plasma, if a resolution of 0.5 Å is assumed. Some prominent lines are listed according to ion and wavelength. The summative processes of bremsstrahlung (B), radiative recombination (L), and two-photon emission (R) are shown. Dashed line represents the total continuous emission (C). (a) $T = 3 \times 10^6 K$; (b) $T = 1.6 \times 10^6 K$; (c) $T = 1.6 \times 10^6 K$; (d) $T = 5 \times 10^6 K$.