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Chapter III.

LINE RADIATION AND SPECTROSCOPY

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III.1 THE FORMATION OF ATOMIC X-RAY LINES

III.1.1 The Strongest X-ray Lines

The temperatures of interest in X-ray astronomy are $T > 3\times10^6$ K, wich corresponds to energies $kT > 200$ eV. We can compare these values to the energy required to strip the last electron from a nucleus of charge Z, $E_I = 13.6\ Z^2$ eV. Here E_I stands for ionization energy, and the expression follows directly from the quantum mechanical description for the one-electron atom or, more simply, from the Bohr model. The two most abundant elements in the universe, H and He, have ionization energies of 13.6 and 54.4 eV, so at temperatures above a few million Kelvin they will be fully ionized and cannot emit characteristic atomic lines. This means that the important lines in cosmic X-ray spectra come from the heavier *trace* elements or *metals*, which will be highly ionized but generally not fully stripped. Metals with cosmic abundances $> 10^{-5}$ that of H (by number) are listed in Table 3.1 together with some of the dominant ionization stages and lines.

For a given element, the strength of a particular line depends on several factors, each of which is discussed in more detail below. The most important factor is the relative population of the requisite ion. This depends on the local thermal and radiation environment and possibly on the thermal history, but in general ions which are relatively harder to ionize can exist over a wider range of temperatures. As for neutral atoms, these more stable ions are those with completely filled electron shells, so it is not surprising that He-like (2 electrons, 1 filled shell) and Ne-like (10 electrons, 2 filled shells) ions dominate. Hydrogen-like (1 electron) ions are also important.

All ions with a given number of electrons form what is called an *isoelectronic sequence*. For example, the He-like isoelectronic sequence includes C V, O VII, Ne IX and Fe XXV. The ions in an isoelectronic sequence have similar internal structure. To first order, the electronic states differ only by scaling factors: for example, energies are larger by a factor Z_{eff}^2 and radii are smaller by a factor $1/Z_{eff}$. In multielectron ions, the effective positive charge (in units of $|e|$) acting on an electron, Z_{eff}, is smaller than the total nuclear charge, Z, due to screening by other electrons. Because of this scaling, the strong lines in the X-ray band from a particular ion generally correspond to strong lines in the optical or UV spectrum of the neutral member of its isoelectronic sequence. For example, the H-like ions have strong Lyman α lines at energies $10.2\ Z^2$ eV, and the He-like ions have strong lines at $21.2\ Z^2$ eV, which correspond to the strong n=2 to n=1 transition at 584 Å in neutral He. The scaling is not precise, however, because of higher order effects, such as relativistic corrections to the energy levels which grow like Z_{eff}^4.

III.1.2 Line Emission

Spectral lines are emitted by an ion that makes a transition from an excited state to a state of lower energy. For most elements of interest, a *state* is best characterized by the quantum numbers LSJM, where L is the total orbital angular momentum, S the total spin, J the total angular momentum and M the projection of J on an arbitrary axis. This is called *LS coupling*. In the absence of an external magnetic field, the energy of the state is independent of M, so transitions usually refer to *levels* labeled by LSJ. The *degeneracy* or *statistical weight* of a level is 2J+1, the number of possible values of M. Levels differing only in J have nearly identical energies (the J dependence is the *fine structure*) and form a *term*. Spectral lines from transitions between the various levels of two terms are called a "multiplet." Despite these precise definitions, the word *state* is often used to describe a level or term.

Because of the various ways in which angular momenta can be added in a multi-electron ion, a given *configuration* of individual electron states, or *orbitals*, can combine to give a variety of possible atomic states. The electron orbitals are specified by the principal quantum number n, and the orbital angular momentum *l*. Therefore, the standard notation indicates both the configuration and the level. For example, the ground level of a He-like ion is $2s^2\ {}^1S_0$ (see Fig. 3.1), an excited level is $1s2p\ {}^3P_3$, and an excited level of a Ne-like ion is $1s^2 2p^5 3d\ {}^1P_1$. Here each orbital is labeled in spectroscopic notation, with principal quantum number n followed by s,p,d,f,g... corresponding to orbital angular momentum *l*=0,1,2,3,4.... When an orbital appears more than once in a configuration, it carries an appropriate exponent called the *occupation number*. The notation often omits orbitals for lower-lying closed shells that do not change during a transition. The total angular momentum is designated by upper-case letters preceded by a superscript that denotes the multiplicity 2S+1 and followed by the subscript J. If the subscript is omitted, the notation refers to the entire term. It is common to refer to "orbitals" as "electrons," as in "a 2p electron," even though the actual wave function cannot assign a particular electron to a given orbital (rather the wave function must be a fully anti-symmetric combination of orbitals often expressed as a *Slater determinant*).

For historical reasons the n=1,2,3,4,... orbitals are also refered to as the K,L,M,N,... shells. This nomenclature is also used for X-ray lines. The K,L,M,N... lines are transitions to the shell with n=1,2,3,4,.... An additional greek letter, α,β,χ,..., denotes transitions with $\Delta n=1,2,3,...$, (e.g. the Kβ line refers to the n=3 to n=1 transition), and numerical subscripts are used to further distinguish between multiplets, if necessary.

Radiative Transitions

The strongest lines correspond to electric dipole transitions (Rybicki and Lightman 1979, pp. 271ff). These are the lowest order terms in a semi-classical perturbation series expansion of the interaction of an ion with an electromagnetic field.

The probability per unit time of a spontaneous downward transition from upper state u to lower state l is given by the *Einstein A coefficient*,

$$A_{ul} = [\omega^3/3\hbar c^3][2|\langle u|er|l\rangle|]^2. \tag{3.1}$$

Here ω is the angular frequency of the emitted photon. Electric dipole transitions can occur only between levels that satisfy the selection rules $\Delta S = 0$, $\Delta L = 0,\pm1$ excluding L=0 to L=0, $\Delta J = 0,\pm1$ excluding J=0 to J=0 (these apply to pure LS coupling), because for all other transitions the matrix element of equation 3.1 vanishes.

Equation 3.1 is similar to the classical electric dipole approximation for the power radiated by an oscillating charge distribution,

$$P = [\omega^4/3c^3]p_o^2, \quad 3.2$$

where p_o is the amplitude of the classical electric dipole moment, $p(t)=\mathrm{Re}[p_o e^{i\omega t}]$. Equation 3.1 can be obtained from 3.2 by noting that for a radiating atom, $P = \hbar\omega A_{ul}$, and by making the correspondence between p_o and $2|\langle u|er|l\rangle|$. When the transition refers to two levels, the matrix element is replaced by an appropriate average over the upper states and a sum over the lower states.

The transition probability is often expressed in terms of the *oscillator strength* f_{ul} (Cowan 1981, p. 404; Tucker 1975, p. 96),

$$A_{ul} = -[2e^2\omega^2/mc^3]\, f_{ul}, \quad 3.3$$

$$= -\alpha^3 f_{ul} [E_{ul}/Ry]\, \omega, \quad 3.4$$

$$\text{so } f_{ul} = -[2m\omega/3\hbar e^2]\, |\langle u|er|l\rangle|^2, \quad 3.5$$

$$= -[1/3]\, [E_{ul}/Ry]\, [|\langle u|r|l\rangle|/a_o]^2. \quad 3.6$$

The (positive) energy difference between the two states is $E_{ul} = \hbar\omega$, Ry is the Rydberg (Ry = 13.5 eV), $a_o = \hbar^2/me^2 \approx 0.5$ Å is the Bohr radius of hydrogen, and $\alpha = e^2/\hbar c \approx 1/137$ is the fine structure constant. The oscillator strength is a dimensionless quantity whose definition is more straightforward when one considers photon absorption accompanied by a transition from l to u: when $f_{lu} = 1$, the quantum mechanical absorption cross section integrated over frequency is identical to that of a classical harmonic oscillator of charge e. In general, f is not far from unity for electric dipole transitions. The minus signs in equations 3.3 and 3.4 reflect the convention that emission oscillator strengths are negative. One of several *sum rules* states that the sum of the oscillator strengths for all possible upward or downward transitions from a given state exactly equals the number of electrons in the atom. In that sense, the aggregate behavior of the electrons in an atom is that of a equal number of classical harmonic oscillators.

We can obtain estimates of the magnitudes of A and f for X-ray lines by considering a hydrogenic ion of charge Z. In equation 3.6, $E_{ul}/Ry \sim Z^2$ while $|\langle u|r|l\rangle|/a_o \sim 1/Z$, so $|f_{ul}|$ is of order unity and is independent of Z. Independence from the strength of the Coulomb force is plausible because the integrated absorption cross section for a classical harmonic oscillator is independent of its restoring force (which does, however, fix the resonant frequency). Evaluating equation 3.4 for the Einstein A coefficient gives

$$A_{ul} \sim 10^9 Z^4 \ s^{-1}. \quad 3.7$$

Non-hydrogenic ions follow the same scaling with Z replaced by an appropriate Z_{eff}. Therefore, the strong X-ray lines of Table 3.1 will typically have spontaneous transition probabilities of 10^{12} to 10^{14} s^{-1}. Correspondingly, an excited state that can decay by electric dipole radiation has a characteristic decay time of 10^{-12} to 10^{-14} s. The relative strengths of lines of a given multiplet follow general rules (which hold to the extent that LS coupling is valid). The strongest lines have $\Delta J = \Delta L$, the sum of the strengths of all lines that originate (or end) on a given level J is $\propto 2J+1$, and the total strength of all lines in a multiplet is $\propto 2S+1$.

After electric dipole transitions, the next highest order terms are the electric quadrupole and magnetic dipole transitions. The transition probabilities for each of these is many orders of magnitude smaller, although in some cases they increase with Z_{eff} relative to the electric dipole term along a isoelectronic sequence. Higher order transitions are observed as *forbidden lines* if they provide the only path for radiative decay of a metastable state and if collisional deexcitations are rare, as is usually the case in low density astrophysical plasmas. In highly ionized atoms, forbidden transitions between levels in the same term are visible in the optical. For example, the 3p $^2P_{3/2}$ to 3p $^2P_{1/2}$ transition of Fe XIV at λ5303 is prominent in the solar corona (hence its designation as a *coronal line*) and has been seen in several galactic supernova remnants and active galactic nuclei.

Transitions that apparently do not obey the electric dipole selection rules readily occur in multielectron atoms for which the eigenstates are not pure LSJM states. An important example is the *intercombination* or *intersystem* line of the He-like isoelectronic sequence (see Fig. 3.1). This 1s2p $^3P_1$ to $1s^2$ $^1S_0$ transition violates the $\Delta S = 0$ rule. It occurs because the true excited eigenstate includes a very small contribution from the 1s2p $^1P_1$ level which can decay to the $^1S_0$ ground state by electric dipole radiation. Another forbidden transition to the same ground state from 1s2s $^3S_1$ occurs by magnetic dipole radiation because of a similar mixing of levels; otherwise this level could only decay by two photon emission, which is of still higher order. The relative strengths of these lines are effective as diagnostics of the emitting plasma (see Section IV.4).

The spectra of high temperature plasmas often show secondary lines at energies only slightly removed from those of strong electric dipole transitions of a given ionization stage. These *satellite lines* involve the same electric dipole transition, but the radiating atom contains one or more additional loosely bound electrons. The extra electrons do not participate in the transition, but they cause a slight shift of the atomic energy levels and hence a shift in the energy of the emitted photon. For example, satellites to the strong 1s2p ^1P - $1s^2$ ^1S transitions in He-like ions are caused by Li-like ions (e.g. $1s2p^2$ ^2D - $1s^22p$ ^2P, which in iron occurs at an energy lower by 0.7%; see Fig. 3.1). Initial states for satellite transitions often follow dielectronic recombination or inner shell excitation (see below), and the relative strengths of satellite lines can be a sensitive plasma diagnostic (see section III.4).

III.1.3 Excitation and Ionization

Collisional Excitation

An inelastic collision of an electron with an ion can leave the ion in an excited state. Classically, the cross section for this process is

$$\sigma_{lu} = \int_0^\infty P_{lu}(r)\, 2\pi r\, dr, \qquad 3.8$$

where $P_{lu}(r)$ is the probability that an electron with impact parameter r will induce a transition from state l to u. The form of the quantum mechanical expression can be obtained from equation 3.8 by replacing the integral over r with a sum over the quantized angular momentum of the scattering electron of velocity v (Aller et al. 1940). Using $mvr = \sqrt{[l_e(l_e+1)]}\hbar$ and $E_e = mv^2/2$ gives

$$\sigma_{lu}(v) = \pi(\hbar/mv)^2\, \Sigma(2l_e+1)\, P_{lu}(l_e) \qquad 3.9$$

$$= \pi(\hbar/mv)^2\, [\Omega_{lu}/(2J_l+1)] \qquad 3.10$$

$$= \pi a_o^2(Ry/E_e)[\Omega_{lu}/(2J_l+1)]. \qquad 3.11$$

In equations 3.10 and 3.11 the sum in equation 3.9 has been replaced by the *collision strength* Ω_{lu} for the transition, which is obtained from the Born approximation by summing the appropriate matrix elements over l_e and averaging over the states of the initial level l (Bell and Kingston 1974). The collision strength is nearly independent of the velocity of the electron, so equation 3.10 implies that $\sigma_{lu}(v) \propto v^{-2}$. For the strong transitions of Table 3.1, Ω_{lu} ranges from 0.005 to 0.15 and is roughly approximated by

$$\Omega_{lu}/(2J_l+1) \approx 4\ (Ry/E_{lu}). \qquad 3.12$$

In a thermal plasma at temperature T, the total collisional excitation rate coefficient R_{lu} is obtained by integrating equation 3.10 over the Maxwell-Boltzmann distribution of electron velocities f(v),

$$R_{lu} = \int_{v_o}^{\infty} v\, \sigma_{lu}(v)\, f(v)\, dv, \qquad 3.13$$

where

$$f(v) = 4\pi[m/2\pi kT]^{3/2} v^2 \exp\{-E_e/kT\}, \qquad 3.14$$

and $mv_o^2/2 = E_{lu}$. We define R_{lu} so that the number of collisions per ion per second in a plasma with electron density n_e is $n_e R_{lu}$. The integration in equation 3.13 gives

$$R_{lu} = 2\pi(\hbar/m)^2\, \Omega'\, \sqrt{[m/2\pi kT]}\, \exp(-E_{lu}/kT), \qquad 3.15$$

$$= 8.6\times10^{-6}\, T^{-1/2}\, \Omega'\, \exp(-E_{lu}/kT)\ cm^3\ s^{-1}. \qquad 3.16$$

Here Ω' is $\Omega_{lu}/(2J_l+1)$ averaged over f(v), but because of the weak dependence on v the averaging does not have a strong effect. A frequently used approximation which relates Ω' to the oscillator strength f_{ul} of the transition (see 3.4 - 3.6) is

$$\Omega' = [8\pi/\sqrt{3}]\, f_{ul}\, [Ry/E_{ul}]\, \langle g(T)\rangle, \qquad 3.16b$$

where $\langle g(T)\rangle$ is a correction factor (Gaunt factor) averaged over the Maxwell-Boltzmann distribution (Tucker 1975, p. 280; Burgess and Summers 1987). It is a weak function of T with typical values of 0.1 - 0.2. Generally, Ω' is corrected upward by 5-10% to account for the contribution of cascades following excitation to higher levels that are otherwise not accounted for.

The exponentials in 3.15 and 3.16 reflect the fact that only electrons with speeds greater than v_o can excite the ion. Using 3.12, typical values of R_{lu} for plasma of $T = 10^7$ to 10^8 K are 10^{-10} to 10^{-12} $cm^3\ s^{-1}$.

Equation 3.15 shows that $R_{lu} \approx m^{-3/2}$, so in general proton collisions will be nearly 100,000 times less effective than electron collisions at causing transitions.

Photoexcitation

Absorption of a photon can excite an ion to an upper level. The photoabsorption cross section is sharply peaked at frequency $\omega_o = E_{lu}/\hbar$ with a width usually determined by the thermal

Doppler motion of the ion, $\Delta\omega/\omega = \sqrt{[2kT/Mc^2]}$, where M is the mass of the ion (Rybicki and Lightman 1979, p. 287). The peak value of the cross section is

$$\sigma_{lu}(\omega_o) = -(\pi e^2/mc)\, f_{lu}\, 2\sqrt{\pi}/\Delta\omega. \qquad 3.17$$

Note that

$$(\Delta\omega/2\pi)\, \sigma_{lu}(\omega_o) = (f_{lu}/\sqrt{\pi})\, [\pi e^2/mc]. \qquad 3.18$$

The second factor on the right of 3.18 is the absorption cross section integrated over frequency for a classical harmonic oscillator of charge e; the first factor gives the quantum mechanical corrections. Equation 3.17 can be rewritten,

$$\sigma_{lu}(\omega_o) = 4\pi^{3/2}\, a_o^{\,2}\, f_{lu}(Ry/E_{lu})(Ry/kT)^{1/2}(M/m)^{1/2} \qquad 3.19$$

Typical values for transitions of interest are ~ 10^{-16} - 10^{-17} cm^2.

Collisional Ionization

An inelastic collision of an electron with an ion can eject one of the bound electrons, leaving the atom in the next ionization state. This process is clearly similar to collisional excitation, with the upper level u replaced by a continuum electron state. Therefore, the cross section σ_{lc} should bear some similarity to σ_{lu} of equation 3.11. A classical estimate of σ_{lc} which has the correct order of magnitude can be obtained from the Rutherford formula for the Coulomb scattering of two charged particles, considering only those trajectories that transfer energies $>E_I$ (Seaton 1962; Bell and Kingston 1974). This gives

$$\sigma_{lc}^{classical} = \pi a_o^{\,2}\, (E_I/E_e)^2\, (E_e/E_I - 1) \text{ for } E_e > E_I. \qquad 3.20$$

Various approximate quantum mechanical calculations have given mixed results when compared to experimental data even for hydrogenic ions, so most computations make use of semi-empirical formulae. The main correction to 3.20 is a factor $\ln(E_e/E_I)$, which changes the behavior at high energies. For multielectron atoms, equation 3.20 must be multiplied by the number of electrons in a shell and summed over shells.

In a thermal plasma, the collisional ionization rate coefficient, R_{col} defined analagously to R_{lu} in equation 3.13, can be approximated by

$$R_{col} = 7\times10^{-11}\, T^{1/2}\, \upsilon F\, (Ry/E_I)^2 \exp(-E_I/kT) \text{ cm}^3 \text{ s}^{-1}, \qquad 3.21$$

where υ is the number of outer shell electrons and F is a factor or order unity (e.g. F = 1.5 for Fe; Seaton 1964; Raymond and Smith 1977) . Applying this to O VII at 10^7 K, for example, gives $R_{col} = 2\times10^{-11}$ cm^3 s^{-1}.

Autoionization

In multielectron atoms, an inelastic collision can transfer to the atom more energy than is required for ionization yet leave the atom in a bound, excited state. This occurs when the collision raises an inner shell electron to a sufficiently high level or when it excites more than one electron. Such a *doubly excited* or *autoionizing* state can decay radiatively, leaving the atom its original ionization stage. Alternatively, it can decay non-radiatively by ejecting an electron, further ionizing the atom; this latter process is called *autoionization*. An example is an atom initially in the $1s^22s^22p$ configuration, which is excited to the $1s2s^22p^2$ configuration. It can radiatively decay to its initial configuration or it can autoionize by ejecting one of the 2p electrons in a transition to $1s^22s^2$. Deexcitation by collision with yet another electron is possible but very rare at the densities of astrophysical plasmas. Autoionization is particularly important for configurations with many inner shell electrons and only one or two outer shell electrons, such as the Li, Be, or Na isoelectronic sequences.

Estimates of the autoionization rate coefficient can be obtained by multiplying R_{lu}, the rate coefficient for collisional excitation of an inner shell electron (eq. 3.16), by a *branching ratio* that gives the fraction of ionizations that actually occur from the autoionizing state. Typical values for Ω' times the branching ratio are 0.1-0.5. Comparing this rate to that of direct collisional ionization (eq. 3.21) shows that the two processes are of similar importance in plasmas at X-ray temperatures. More precise computations of the autoionization rates of highly ionized species are difficult. Although consideration of equation 3.16 and computations for hydrogenic ions predict no strong dependence of the rate on Z_{eff}, experiments show that simple extrapolation along an isoelectronic sequence is not possible. Imprecise autoionization rates have been a major source of uncertainty in calculations of the ionization equilibrium in thermal plasmas (Shull and van Steenburg 1982).

Photoionization

Photoionization is related to photoexcitation in the same way that collisional ionization is to collisional excitation. The cross section has a form similar to that of equation 3.17

$$\sigma_{lc}(\omega) = (\pi e^2/mc)\, 2\pi\, df_{lc}/d\omega. \qquad 3.22$$

The last factor is the oscillator strength per unit frequency for transitions to the continuum from level l. An approximation for this can be obtained by extrapolating f_{lu} for hydrogenic ions, which gives

$$\sigma_{nc}(\omega) = (64\pi/3\sqrt{3})\, \alpha a_o^2\, (n/Z^2)\, (E_{In}/E_{nc})^3, \qquad 3.23$$

$$= 7.9\text{x}10^{-18}\, (n/Z^2)\, (E_{In}/E_{nc})^3\ \text{cm}^2, \qquad 3.24$$

with $E_{nc} > E_{In}$.

Here n is the principal quantum number of the level, E_{In} is the ionization energy and $E_{nc} = \hbar\omega$. More exact expressions include a correction factor (or *Gaunt factor*) $g(E_{nc},n)$, which alters slightly the energy dependence (Cowan 1981; Tucker 1975, p. 241).

X-rays absorption by neutral or slightly ionized atoms will generally result in the ejection of an inner shell electron, leaving an ion in a highly excited autoionizing state. In this case, the subsequent decays often eject additional electrons. Autoionization following a vacancy in the innermost shells of an atom is called the *Auger* process. For neutral atoms of intermediate Z, like C, N, and O, >99% of the inner shell ionizations are followed by emission of a second, Auger electron. In higher Z atoms, such as Fe, the increased probability of radiative transition (see 3.7) reduces the likelihood of Auger autoionization.

III.1.4 Recombination

Radiative Recombination

The process of electron capture by an ion to produce a photon and an atom in the next lower stage of ionization is the inverse of photoionization. The cross sections for two such competing processes can be related to one another by using the principal of *detailed balance*. This exploits the fact that an ion in true thermodynamic equilibrium with a radiation field must have equal net rates of destruction by photoionization and of formation by radiative recombination. The relative population of the ionization stages is set by thermodynamics (the Saha equation) as is the radiation field (the Planck black body formula). Using these, detailed balance fixes a relation between the two cross sections that must hold whether or not the ion is in equilibrium (this is the method used by Einstein to relate the A and B coefficients for radiative transitions):

$$\sigma_{cl}(E_e) = W\,\alpha^2\,(E_e+E_{Il})^2/(RyE_e)\,\sigma_{lc}(\omega). \qquad 3.25$$

Here $\sigma_{cl}(E_e)$ is the radiative recombination cross section for electrons of energy E_e, E_{Il} is the ionization energy for level l of the recombined ion, $\omega = (E_e + E_{Il})/\hbar$, W is the ratio of the statistical weight of level 1 to that of the recombining ion, and σ_{lc} is given by equation 3.22. Using the approximation of 3.23 gives an expression valid for recombination to the nth shell of hydrogenic ions

$$\sigma_{cn}(E_e) = (32/3\sqrt{3})\,n\alpha^3\,\pi a_o^2\,\{E_{In}^2/[(E_e+E_{In})E_e]\}, \qquad 3.26$$

$$= 2.1\text{x}10^{-22}\,n\,\{E_{In}^2/[(E_e+E_{In})E_e]\}\ \text{cm}^2. \qquad 3.27$$

Using 3.26 and 3.27, we can compute the radiative recombination coefficient for a thermal plasma, defined analagously to equation 3.13 (Cowan 1981, p. 547; Tucker 1975, p. 218):

$$\alpha^r_{cn} = 5.2\text{x}10^{-14}\,n\,(E_{In}/Ry)^2\,(Ry/kT)^{3/2}\exp(E_{In}/kT)\,\varepsilon_1(E_{In}/kT)\ \text{cm}^3\ \text{s}^{-1}, \qquad 3.28$$

$$\text{with } \varepsilon_1(x) = \int_x^\infty e^{-y}\,dy/y\ \ (\text{e.g. } \varepsilon_1(1) = 0.22). \qquad 3.29$$

For example, $\alpha^r_{cn} = 2.5\text{x}10^{-13}$ cm^3 s^{-1} for $E_{In}/k = T = 10^7$ K and n = 1.

At densities above 10^{18} cm^{-3} three body recombination is also important. This is the "inverse" of collisional ionization; the energy released by the recombining ion is carried off by the additional electron.

Dielectronic Recombination

Dielectronic Recombination is the inverse of autoionization. In this case a free electron is captured by an atom. However, the excess energy is not radiated, as it is in radiative recombination, but appears as an additional excitation of the atom to an autoionizing state. Using the example from our description of autoionization, an ion in the $1s^22s^2$ configuration makes a radiationless capture of an electron, which leaves it in the $2s2s^22p^2$ configuration. Then its fate is the same as described earlier: it can autoionize, returning to its initial configuration, or it can radiatively deexcite to give $1s^22s^22p$, which accomplishes the recombination.

Although the rate of formation of an autoionizing state is relatively independent of the degree of ionization, dielectronic recombination becomes increasingly important relative to radiative recombination along an isoelectronic sequence. This is because the probability for radiative decay from this highly excited state increases rapidly with Z_{eff} (see for example eq. 3.7). In X-ray emitting plasmas, dielectronic recombination is generally the dominant process for non-hydrogenic ions. For example, for Fe XVII in a 10^7 K plasma the rate for dielectronic recombination is ~6 times the radiative rate. However, at densities $>10^{12}$ cm^3 dielectronic recombination will be suppressed because collisional ionization of the intermediate, autoionizing state reduces the branching ratio of radiative decay.

A rough approximation for the dielectronic recombination rate coefficient for ions with Z_{eff} ~ 5 - 15 in a thermal plasma at temperature T is

$$\alpha^d = 5\times10^{-11}\,(kT/Ry)^{-3/2}\,Z_{eff}^{5/2}\exp(E_I/kT), \qquad 3.30$$

(Cowan 1981, pp. 549ff; Tucker 1975, p. 223). Here E_I is the ionization energy of the recombined ion. Equation 3.30 gives $\alpha^d \sim 10^{-11}$ $cm^3\,s^{-1}$ for reasonable values of the parameters.

The radiative decay of an autoionizing state following dielectronic recombination can give rise to *satellite lines*. These are lines at energies very close to those from radiative transitions of singly excited states of the unrecombined ion. An example is the dielectronic recombination of He-like ions to the autoionizing state of the Li-like ion 1s2pnl. This can decay to the still excited state $1s^2nl$. The emitted photon will have an energy close to that of the corresponding He-like transition 1s2p - $1s^2$; the difference in energies comes from the perturbation of the energy levels by the "spectator" nl electron. For n = 2 or 3, the satellite lines can generally be resolved and often give important diagnostic information about the emitting plasma. For larger n or in lower resolution studies the satellite line simply adds to the intensity of the He-like line. The relative increase in the dielectronic recombination rate with Z_{eff} means that the relative strength of satellite lines also increases along an isoelectronic sequence (see III.4. for further discussion).

III.2 CORONAL PLASMA

III.2.1. Fundamental Properties

Astrophysical X-ray line emitting plasmas cover a vast range of density, size and age. Table 3.2 lists some typical values for these parameters (see also more detailed chapters on each of these systems). How they are determined is described in Section III.4 below. From the point of view of atomic physics, astrophysical plasmas can be divided into two broad categories: *thermal* plasmas and *photoionized* plasmas. Thermal plasmas are discussed in this section, and photoionized plasmas in section III.3.

Despite the range of parameters in Table 3.2, nearly all the *thermal* plasmas have common characteristics. First, most have sufficiently low particle densities that collisional excitation occurs at a much lower rate than radiative decay. We can see this by comparing the relevant time scales. Equation 3.16 gives a collisional excitation time scale of 10^{10} n_e^{-1} to 10^{12} n_e^{-1} s, whereas the radiative decay time scale for electric dipole transitions is 10^{-12} to 10^{-14} s (equation 3.7). Thus decay proceeds at a much faster rate than excitation in any plasma with density less than $n_e \approx 10^{22}$ cm^{-3}, a limit that exceeds the typical values for even the densest regions in the solar corona. Even an ion excited to a metastable state typically has time to decay radiatively by a higher order transition before being disturbed by a collision. This means that in most astrophysical plasmas, an ion is nearly always in its ground state, and when it is excited, it generally decays by photon emission. With homage to the solar prototype, such systems are called *coronal plasmas* (see Pallavicini 1988).

Another characteristic of such a relatively low density plasma is that it is *optically thin*, meaning that a radiated photon has very little chance of interacting before it leaves the plasma. For a continuum X-ray photon, the most important absorption process is photoionization of intermediate Z elements, which has a cross section near threshold given by 3.24. In a plasma, the effective cross section depends on the elemental composition and the degree of ionization. Figure 3.1 shows the effective absorption cross section $\sigma_{abs}(E)$ for photons of energy E normalized per hydrogen atom for a mix of elements with cosmic abundances at various temperatures. For $T \geq 10^5$ K, $\sigma_{abs}(E) \leq 10^{-22}$ cm^2 for all E. The column densities, n_eL, of all the thermal plasma sources in Table 3.2 are $\leq 10^{21}$, so most continuum photons will escape.

The thermal plasma sources have greatest opacity for line photons emitted by atomic transitions to the ground state of an abundant ion, because these generally have just the right energy to excite another ion from its ground state. Equation 3.19 gives a typical peak cross section of 10^{-16} cm^2 for photoabsorption in a strong line. To compare with the column densities of Table 3.2, the cross section must be multiplied by the abundance of the ion relative to hydrogen (Table 3.1). For a strong line of oxygen, for example, the peak cross section could be as large as ~ 10^{-19} cm^2 at some temperatures, so some astrophysical systems will not be optically thin to all line photons. For strong transitions, however, the absorbing atom will usually just reemit a photon in the same line; the net result of the absorption is merely a nearly elastic scattering of the original photon. This phenomenon is *resonance scattering*, and lines involving transitions to the ground state are *resonance lines*. Resonance absorption can alter the emitted spectrum if the atom has alternative transitions from the upper level (i.e. in a hydrogen like ion a Lyman β n = 3 to 1 line can be converted into a Balmer n = 3 to 2 line and a Lyman α n = 2 to 1 line). Furthermore, resonance scattering causes a photon to random walk through the plasma, which temporarily traps it (so-called *resonance trapping*), increasing its effective path length and making it more susceptible to continuum absorption. In non-spherical geometries, such as in solar coronal loops, resonant scattering can alter the angular distribution of the line emission. This last effect has been seen in the sun.

III.2.2. Statistical Equilibrium

Ionization Balance

Coronal plasmas are far from being in true thermodynamic equilibrium. In thermodynamic equilibrium the relative populations of the excited states of an atom are derived from the Boltzmann distribution; in a coronal plasma atoms are nearly always in their ground states. In thermodynamic equilibrium the radiation field is that of a black body; in a coronal plasma the radiation, mostly *bremsstrahlung* and lines, has many orders of magnitude less energy density and barely interacts with the matter.

In a steady state, the relative population of the ionization stages of an ion in a coronal plasma can be deduced from the principle of statistical equilibrium: the total number of ionizations to the next higher stage must equal the total number of recombinations from that higher stage (this approach can be extended to encompass multiple ionization such as the Auger process),

$$n(X^r)\, R_{X^r}(T) = n(X^{r+1})\, \alpha_{X^{r+1}}(T), \qquad 3.31$$

$$n(X^r)/n(X^{r+1}) = \alpha_{X^{r+1}}(T)/R_{X^r}(T). \qquad 3.32$$

Here $n(x^r)$ is the density of element X in ionization stage r, R(T) is the collisional ionization rate coefficient and $\alpha(T)$ is the recombination rate coefficient. We assume that the electrons in the plasma have a Maxwellian velocity distribution of temperature T (see Section III.2.3). In general, R is dominated by direct collisional ionization (see eq. 3.21); autoionization can also be important, especially for multielectron ions with only one or two outer shell electrons. Photoionization is generally negligible if the column density is $< 10^{22}$ cm^{-2}. Similarly, α includes contributions from radiative recombination (3.28) and dielectronic recombination (3.30); the latter can dominate for some ions.

Figure 3.2 shows the ionization fractions at equilibrium for oxygen and iron in a coronal plasma as derived from equation 3.32. In general, the ion of maximum abundance in a coronal plasma has ionization energy $E_I \sim$ 1-3 kT. Coronal plasma is less highly ionized than plasma at thermodynamic equilibrium at the same temperature, in which the most abundant ions typically have $E_I \sim$ 3-10 kT.

Line Emission

Because the radiative time scale is generally much shorter than the collisional time scale in a coronal plasma, the strength of a line from a given transition depends almost entirely on the rate at which the upper level of that transition is populated. For most strong lines this rate is dominated by collisional excitation. Thus the power emitted per unit volume in a line from the l-u transition of ionization stage r of element X is

$$P_{lu} = n_e n(X^r)\, E_{lu}\, R_{lu} \text{ erg cm}^{-3}\text{ s}^{-1}, \qquad 3.33$$

where R_{lu} the collisional excitation rate coefficient from equation 3.16. Equation 3.33 is often rewritten as

$$P_{lu}/n_e^2 = [n(H)/n_e]\,[n(X)/n(H)]\,[n(X^r)/n(X)]\, E_{lu}\, R_{lu} \text{ erg cm}^3\text{ s}^{-1}. \qquad 3.34$$

The first factor on the right hand side is the number of hydgrogen atoms per electron, which is $\sim[1 + 2n(He)/n(H)]^{-1} = 0.85$ for cosmic abundances. The second factor is the abundance of element X relative to hydrogen and the third is the fraction of ions of X in the r^{th} ionization stage. The ionization fraction is the most temperature sensitive factor in equation 3.34, so the maximum line emission occurs at the temperature at which the ion is most abundant. For strong lines, peak values of P_{lu}/n_e^2 are $\sim 10^{-24}$ erg cm^3 s^{-1}. Peak values for the strongest lines are given in Table 3.1)

Cascades from excitations to higher levels and from radiative and dielectronic recombination can also populate the upper level of a transition and contribute to line emission. Generally these processes are important only for higher order transitions such as forbidden and intercombination lines in He like ions (see III.4).

Continuum Emission

Thermal *bremsstrahlung* (see Chapter I) is the main source of contiuum photons, although there are also contributions from radiative recombination and two-photon decays from levels with no single photon decay path. For example, at 10^7 K *bremsstrahlung* contributes ~90% of the continuum at ~ 1 keV and ~50% at 5 keV, most of the remainder is from radiative recombination. Two-photon emission is only important around 0.5 keV for T ~ 10^6 K. At 10^8 K *bremsstrahlung* dominates at all energies. It is possible to modify the *bremsstrahlung* Gaunt factor of equation 1.xx to approximate all continuum emission processes,

$$G(T) \sim 5.6\times10^{13}\, T^{-2} + 6.3\times10^{-5}\, E^{-0.34}\, T^{0.2}, \qquad 3.35$$

(Mewe and Gronenschild 1981, eq. 47).

Calculated Spectra

Computations of the total line and continuum emission from an isothermal coronal plasma with cosmic abundances have been carried out by several authors (e.g. Raymond and Smith 1977, Mewe et al. 1985) who are periodically update their computer codes to incorporate improvements in the atomic physics (Raymond 1988). Figure 3.3 gives an example of the calculated spectrum of a coronal source at 10^7 K. For each significant emission line, the models give the emissivity Plu/n_e^2 (eq. 3.34) at discrete temperatures from 10^6 to 10^8 K (figure 3.34). Model spectra can be integrated to give the total emissivity, $\Lambda(T)$ of a thermal plasma, and such a *cooling function* is shown in Figure 3.4. As the figure shows, the intgrated line emission is the major component of the radiation for all T < 10^7 K, and at some temperatures, only a few lines dominate. Observers using detectors with moderate or low spectral resolution, like a silicon detector or proportional counter, fold their instrument response functions with the results of these models to create libraries of fitting functions that they apply to the data. High resolution spectrometers, like Bragg spectrometers, gratings, or calorimeters, measure individual line strengths which can be compared directly to the model calculations (Holt 1989, Canizares 1989).

III.2.3 Departures from Equilibrium

There are three equilibria relevant to coronal plasmas, each of which can affect the emitted spectrum. First, there is the equilibration of energy amongst the electrons, leading to the Maxwell Boltzmann velocity distribution. Second, there is the equipartition of energy between protons and electrons, and third there is the equilibration between ionization and recombination that leads to the steady state distribution among ionization stages. In many cases, the time scales for establishing the various equilibria are long compared to the age of the astrophysical object or to the time scale for impulsive heat input, as in solar flares.

Non-Maxwellian Electron Distribution

Energy exchange between electrons by elastic Coulomb scattering has the shortest time scale of the processes considered here. It is given roughly by $0.011\, n_e^{-1}\, T^{3/2}$ s = $3\text{x}10^8\, n_e^{-1}$ s for T = 10^7 K (Spitzer 1978, pp. 20ff). Comparison with Table 3.2 shows that in all astrophysical plasmas this time scale is short compared to the age of the system. For this reason, nearly all computations assume that the electrons have a Maxwellian velocity distribution.

In situ measurements of electron velocities in the solar wind and in planetary magnetospheres do show departures from Maxwellian distributions, but even if similar departures occur in the systems of Table 3.2, they will probably not alter significantly the line emitting characteristics of the plasma. It is thought that departures from Maxwellian distributions occur where there are large gradients in temperature and density, because the longer mean free path for high energy electrons enables them to migrate from hotter to cooler regions thus distorting the velocity distribution in the cooler region. In line emission, the electron velocity distribution enters as a weighting factor in the calculation of the mean interaction rate coefficients of section III.1 (see Eq 3.13). Sample computations show that distortions of the distribution have little effect on the recombination rates and affect ionization (or excitation) rates only when E_I > 8-10 kT, whereas in ionization equilibrium E_I ~ 1-3 kT (Owocki and Scuder 1983).

Unequal Electron and Proton Temperatures

If a plasma is heated by a shock wave, it is possible that the protons will be raised to a higher temperature than the electrons. Such a situation may occur in supernova remnants, where T_p/T_e could be as high as m_p/m_e just inside the shock front (see Section VII.E). It may be that collective plasma processes rapidly equilibrate these temperatures, otherwise equilibration will only occur on the time scale of electron-proton Coulomb collisions, which is ~$10^{11}\, n_e^{-1}$ s (McKee and Hollenbach 1980). All the processes of section III.1 depend on the electron temperature, so models of line and continuum emission computed with T = T_e can be used in any case.

Non-Equilibrium Ionization

The time scale for ionization of an atom to its next ionization stage is $(n_e R_{col})^{-1}$ which, for example, is ~$5\text{x}10^{12}/n_e$ s for OVII at 10^7 K (see Eq. 3.21). Comparison with the parameters of Table 3.2 shows that many supernova remnants are too young to have achieved ionization equilibrium and that solar flares will approach equilibrium on a time scale accessible to observation. For these systems, the assumption of ionization balance of Eq. 3.32 is not justified, and it is necessary to follow in detail the time dependence of the ionization state of the plasma.

The time dependent equation for the ionization fraction $f(X^r)$ of the r^{th} ionization stage of element X,

$$f(X^r) = n(X^r)/n(X), \qquad 3.36$$

is given by

$df(X^r)/dt = n_e\{R_{X^{r-1}}f(X^{r-1} - (R_{X^r} + \alpha_{X^r})f(x^r) + \alpha_{X^{r+1}}f(x^{r+1})\},$ 3.37

It is convenient to substitute $\tau = tn_e$ in this expression, which removes the explicit dependence on n_e. An element of charge Z will have Z+1 such coupled equations with the added condition that $\Sigma f = 1$. These can be integrated numerically (Mewe and Schrijver 1978) or solved by a more general eigenvalue technique (Hughes and Helfand 1985).

Examples of the temporal evolution of the ionization fractions for O and Fe are given in Figure 3.5, assuming impulsive heating of the electrons to 10^7 K at $\tau = 0$. The figures show that the progression through subsequent ionization stages "stalls" at stages with closed electron shells (the isoelectronic sequences of the inert gases) such as He-like O VII and Ne-like Fe XVII. This means that the lines from these stages will be anomalously strong in plasmas which are still ionizing. Models of the emission vs. time from supernova remnants, for example, would incorporate these ionization fractions in Eq. 3.34.

The cooling of a plasma can also cause departures from ionization equilibrium. However, this is generally not a problem for plasmas that are cooling by emission of radiation until $T \leq 10^6$ K (Edgar and Chevalie 1986). We can see this by comparing the radiative cooling time of a plasma with the atomic recombination time scale. The cooling time scale is

$$t_{cool} = 3(n_e + n_p)kT/2 \div n_e^2\Lambda(T), \quad 3.38$$

$$\approx 3kT/n_e\Lambda(T), \quad 3.39$$

where we have divided the energy density by the radiation per unit volume, approximated $n_p = n_e$ and ignored the small amount of thermal energy in heavier elements. The cooling function Λ is given in Figure 3.5. The recombination time scale is

$$t_{rec} = [n_e\alpha^r(T)]^{-1}, \quad 3.40$$

with the recombination coefficient α^r given by eq. 3.28 or 3.30. At $T = 10^7$ K, $t_{cool} \sim 50\ t_{rec}$, so recombinations can easily keep up with the cooling rate. This means that the ionization fractions will maintain nearly their equilibrium values at each temperature as the plasma cools. This is reasonable because the the dominant contribution to the cooling function for $T < 10^8$ K is the line emission from the very ions in question, and it has been verified by numerical calculations using Equation 3.37. As a result, the line emission from cluster cooling flows, for example, can be found by an apporpriate superposition of equilibrium models (see section VII D). Plasmas that cool more rapidly, such as by adiabatic expansion, could be overionized and would require a fully time dependent models as do plasmas with $T \leq 10^6$ K.

In addition to effects on the ionization fractions themselves, departures from ionization equilibrium can alter the relative strengths of some of lines from a given ionization stage. This occurs for the forbidden and resonance lines of He-like ions, for example, because population of the upper levels depends on recombinations as well as collisional excitation and the relative importance of these processes changes when the plasma is either under-ionized or over-ionized for its electron temperature. Thus these line ratios become diagnostics for non-equilibrium inonization (see III.4).

III.2.4 Moderately Thick Plasmas

When the column density in a plasma is $\geq 1/\sigma_T$, where σ_T is the Thompson scattering cross section (equation 2.xx), electron scattering will alter the spectrum of the emitted radiation.

The distortion of the continuum spectrum is described in Chapter II.2(?). A line photon that is Compton scattered by an electron in the emitting plasma will typically have its energy altered by an amount

$$\Delta\omega/\omega \sim (2kT/mc^2)^{1/2} = (1/16)(T/10^7K)^{1/2}, \qquad 3.41$$

(see equation 2.xxx). Equation 3.41 shows even a single scattering is sufficient to completely remove the photon from the line, so that the effective line emitting volume of the source is limited to a thin layer near the surface from which the photons can escape freely. If one considers only electron scattering, the thickness of this layer is $\sim(4n_e\sigma_T)$.

Resonance lines are even more susceptible to diminution by Compton scattering because of the resonance trapping phenomenon described in III.2A. The importance of resonance trapping depends on a parameter which is roughly the ratio of the electron scattering to resonance scattering mean free paths

$$\beta = (n_e\sigma_T)/(\pi^{1/2}n(X^r)\sigma_{lu}(\omega_o)), \qquad 3.42$$

$$\sim 4x10^{-9}/(n(X^r)/n(H)), \qquad 3.43$$

(Felten et al. 1972). In 3.43 a typical value of 10^{-16} cm^2 has been used for $\sigma_{lu}(\omega_o)$, the resonance absorption cross section at the line center (see 3.19). In general, a strong resonance line will come from an ion with $n(X^r)/n(H) \sim n(X)/n(H)$, so for all reasonable abundant elements β will be small and resonance trapping is important. A proper treatment of resonance trapping requires consideration of the line profile, because resonance scattering can shift the energy of a photon from the peak of the line to the wings where the absorption cross section is smaller and the mean free path larger. The net result is that resonance trapping reduces the thickness of the layer from which line photons can emerge from the plasma without suffering a Compton scattering by a factor which is approximately $\sim 0.6/\ln(\beta)$ for $\beta < 0.1$. Thus resonance trapping can significantly weaken the resonance lines relative to lines from forbidden transitions, which are not trapped.

In sources with moderate column densities photoionization can be important or even the dominant ionization process. This increases the equilibrium ionization stage at a given temperature. Like the radiative transfer, the ionization balance calculations must take account of the geometry of the source.

III.3 PHOTOIONIZED PLASMA

III.3.1. Fundamental Properties

When a plasma is subjected to strong X-radiation from an external source, its properties differ considerably from those of a coronal plasma. These conditions occur in the stellar winds around X-ray binaries and in emission line clouds near active galactic nuclei, for example. In such plasmas photoionization is more important than collisional ionization and the ionization struture depends in detail on the spectrum of incident radiation and the geometry of the system. The simplest case is a plasma that is optically thin to both photoabsorption and Compton scattering. Here each ion is assumed to see the full spectrum of the external source at the appropriate flux for its distance but unmodified by absorption in the intervening plasma. Furthermore, in this case the radiation emitted at all points by the plasma itself is assumed to escape freely. However, the observed spectra of X-ray binaries, for example, often show low energy cut-offs greater than those expected from absorption by interstellar material, indicating that photoabsorption in the circum-source material is important. Because the Thompson cross section (see II.xx) is generally smaller than photoionization cross sections (eq. 3.24), plasmas can be optically thick to photoabsorption but still thin to Compton scattering. Such plasmas are treated in the *nebular approximation*, named after the similar treatment of planetary nebulae ionized by ultraviolet radiation (the thin plasma approximation is itself a special case of the nebular approximation). Some X-ray binary spectra also show iron lines broadened by Compton scattering, suggesting that the surrounding plasma is thick to electron scattering as well. In either case a description of an optically thick plasma involves a self-consistent treatment of radiative transfer, ionization balance and the balance between heating and cooling.

III.3.2 Ionization Balance and Temperature

In *photoionized plasmas* the ionization balance equation (3.31) must be modified to include the effects of the radiation field (Kallman and McCray 1982):

$$n(X^r)[\zeta(\vec{r},X^r) + n_e R_{X^r}] = n(X^{r+1}) n_e \alpha_{X^r}(T). \qquad 3.44$$

Here $\zeta(\vec{r},X^r)$ is the photoionization rate for ionization stage r of element X at the location $\vec{r}$ in the plasma, which is obtained by integrating the appropriate photoionization cross section σ_{lc} (see eq. 3.22) over the local radiation flux, j_ν (erg cm^{-2} s^{-1} Hz^{-1}),

$$\zeta = \int_{\nu_I}^{\infty} j_\nu \sigma_{lc}(h\nu) d\nu / h\nu. \qquad 3.45$$

The lower limit of integration is the threshold frequency for photoionization, $\nu_I = E_I/h$.

In the optically thick case, eq. 3.44 must be coupled to a radiative transfer equation and solved by iteration. The recombination coefficient α may also depend on density (see section III.1.2): for $n_e > 10^9$ cm^3, dielectronic recombination is suppressed and for $n_e > 10^{18}$ cm^{-3} three-body recombination becomes important for intermediate Z ions (but not for iron). In the optically thin case one can neglect all these complications and also assume that collisional ionization is negligible, which gives

$$n(X^r)/n(X^{r+1}) = n_e \alpha_{X^{r+1}}(T)/\zeta(\vec{r},X^r). \qquad 3.45$$

The temperature of the plasma is computed by balancing all the relevant heating and cooling mechanisms. This situation differs from that in coronal plasmas, which are assumed heated by some external process, such as a strong shock, and for which the cooling times are always much longer than the time scales for all other atomic processes. In the photoionized plasmas the radiation supplies heat to the ions through photoabsorption and to the electrons through Compton scattering. Cooling is by bremsstrahlung, line emission, and inverse Compton scattering (also called Compton cooling). The net energy transfer from Compton scattering is

$$n_e\Gamma_C = n_e F\,[\sigma_T/mc^2][4kT - h<\nu>]\ \text{erg cm}^{-3}\text{s}^{-1}, \qquad (3.46)$$

where F is the local radiation flux integrated over all frequency, σ_T is the Thompson cross section (see II.xx) and $h<\nu>$ is the mean photon energy for the given spectrum. This expression neglects relativistic effects, so it is valid only when the photon energies are « mc^2 = 511 keV.

In optically thin plasmas and for a given spectral shape of the incident radiation, the ionization structure and temperature at a given point in the plasma depend on a single *ionization parameter*, which for the usual geometry of plasma at a distance D from a point X-ray source of luminosity L is given by

$$\xi = L/n_e D^2, \qquad (3.47)$$

The value of ξ is proportional to the number of photons per particle in the plasma.

For intermediate spectra, such as power laws $j_\nu \sim \nu^{-\alpha_{s10<n>}}$ with $\alpha_{s10[n]} \sim 1$ or thermal spectra with characteristic temperatures of ~10 keV, the results of detailed calculations show that $T \sim 10^4$K for $\log(\xi) < 1$ and rises monotonically with ξ to a maximum of $\sim 10^7$K for $\log(\xi) >= 4$ (ξ is in cgs units). At these highest values of ξ, the temperature is set by the competition between Compton heating and Compton cooling; i.e., the temperature is such that the right hand side of equation 3.46 nearly vanishes, so $T_{max} \sim h<\nu>/4k$. Thus the maximum temperature depends on the radiation field, is nearly independent of the properties of the plasma, and is nearly independent of ξ for $\log(\xi) > 4$. For example, for a power law spectrum with $\alpha = 1.0$ that extends from E_{min} to E_{max}, eq. 3.46 gives $T_{max} \sim (1/4k)\,[E_{max}/\ln(E_{max}/E_{min})]$. For a thermal spectrum with temperature T_{rad}, $T_{max} \sim T_{rad}/4$.

For harder spectra (i.e., a power law with smaller $\alpha_{s10[n[}$), the transition between low and high temperatures is more abrupt, occuring over a narrower range of ξ. If the incident spectrum is sufficiently hard (i.e., $\alpha_{s10[n]} \leq 0.75$), the T vs. $\log(\xi)$ relation is triple valued. That is, for intermediate values of $\log(\xi)$, the photoionized plasma can have either $T \sim 10^4$K or $T \sim T_{max}$; an intermediate temperature is formally allowed but plasma at this temperature is thermally unstable and will quickly heat or cool to one of the other two values. Thus material subjected to a sufficiently hard X-ray flux can have two phases of equal pressure: denser clouds at the lower temperature embedded in a hotter, more diffuse medium. The denser clouds may be reponsible for the optical and UV lines seen in quasar spectra (Krolik et al. 1981, Gilbert et al. 1983).

The degree of ionization of a given element increases with $\log(\xi)$, or decreases with distance from the source. This is because photoionization cannot compete with recombination in the more dilute radiation field far from the source. In thin plasmas, a characteristic value of ξ at which a given ion X^r will be found is

$$\xi_{thin}(X^r) = 4\pi\,\alpha_{X^r}(T)/\zeta(\vec{r},X^r). \qquad (3.48)$$

For $\xi < \xi_{thin}(X^r)$ element X will be less ionized than the r^{th} ionization state. Equation 3.48 is obtained by equating the recombination and ionization rates for the ion, evaluated at an appropriate temperature. Higher ionization stages have larger recombination rates, smaller ionization cross sections and higher ionization thresholds, (see equations 3.24 and 3.28) all of which make $\xi_{thin}(X^r)$ in equation 3.48 increase strongly with Z. For example, in a plasma illuminated by a thermal spectrum with kT ~ 10 keV, hydrogen is fully ionized for $\log(\xi) > -1.0$ and He for $\log(\xi) > 1.0$. Heavier elements become progressively more ionized as ξ increases (see Fig. 3.7). For example, OVII predominates at $\log(\xi) \sim 1.5$ and O VIII at ~ 1.8. Typically a given ionization stage persists over an interval $\Delta\log(\xi) \sim 0.5$. At $\log(\xi) = 4.$, there is some H-like iron, Fe XXV, but the intermediate Z elements are fully stripped. Using eq. 3.47, these results show that an active galaxy nucleus with an X-ray luminosity of 10^{45} ergs s^{-1} will almost fully ionize the interstellar medium of the host galaxy (for which n ~ 1 cm^{-3}) to a distance of several kpc, and H will be ionized within 30 kpc.

In plasmas that are optically thick to photoabsorption but not to Compton scattering, it is the depletion of ionizing photons rather than their geometrical dilution that limits the regions of ionization. The characteristic value of ξ below which element X is less ionized than the r^{th} stage is found by equating the total flux of ionizing photons with the number of recombinations per second within the ionized region. Approximating the photon flux by L/E_I and the number of recombinations by $(4\pi/3)\, R^3\, n_e^2\, A_i\, \alpha(X^r,T)$, and using 3.47 gives

$$\xi_{thick}(X^r) = [4\pi A_X \alpha(X^r,T) E_I/3]^{2/3}\, [Ln_e]^{1/3}. \qquad 3.49$$

Here A_X is the abundance of element X relative to H. Equation 3.49 overestimates the size of the ionization zone because the absorption by the inner regions of plasma (high ξ) partially shields the outer parts. But equation 3.49 shows that in addition to ξ, optically thick photoionized plasmas are characterized by the parameter Ln_e (Kallman and McCray 1982). Slightly ionized elements are affected by optical depth for $Ln_e > 10^{40}$ erg s^{-1} cm^{-3}. For larger Ln_e, even highly ionized species are affected. One can show that the column density for a given ion scales as a fractional power of Ln_e.

In an optically thick plasma, the ionization structure becomes more stratified, with more sharply defined zones of $\Delta\log(\xi)$ within which a given ionization stage prodominates. Thus in a point source surrounded by gas one obtains so called *Stromgren spheres* or shells of decreasing ionization with increasing radius just as in an HII region ionized by the ultraviolet continuum from a hot star. The sharp boundaries of the ionization zones can be understood as follows: as the ionizing photons are depleted, the column density of the absorbing ion begins to increase, which further depletes the supply of photons, etc.

Comparing the ionization state of a photoionized plasma with that of the coronal plasmas of section III.2 shows that a photoionized plasma is very significantly over-ionized for its temperature. Whereas the ionization energy of the dominant ion E_I ~ 1-3 kT in a coronal plasma, in a photoionized plasma E_I ~ 0.001 - 0.02 kT for the X-ray emitting ions (recall that in true thermodynamic equilibrium E_I ~ 3 - 10 kT).

III.3.3 Emitted Spectrum

Because a photoionized plasma is severely over-ionized, or equivalently has a very low temperature for its ionization state, collisional excitation is negligible (note the exponential in eq. 3.16). Therefore, the X-ray emission lines are largely due to cascades following radiative recombination. In general, the continuum will be dominated by the external source. Figure 3.8 shows two examples, one for a thin plasma and one for a plasma that is optically thick to photoabsorption but not to Compton scattering. In the latter, the absorption due to partially ionized elements has removed most of the low energy photons. Note that many of the emission lines appear near the low energy cut off in the continuum. This is because, for a given Ln_e, the ionization stages of a given Z_{eff} will have column densities corresponding to an optical depth near unity. The spectrum will be cut off below the corresponding ionization energy $E_I \sim Z_{eff}^2 Ry$. But these ions will also provide most of the emission lines at energies $\geq E_I$. Lines from lower ionization stages, which have even larger column densities, will be absorbed; higher ionization stages have smaller column densities and therefore their lines are weaker.

The K lines of iron near 7 keV are strong in all the optically thick models, in contradiction to the argument just presented (Hatchett et al. 1976). The lines are produced by fluorescence (see below) in all stages of iron, even neutral FeI, and they escape freely because even the "thick" plasmas are generally transparent at such high energies. For plasmas with low energy cut-offs of several keV, 5% of the absorbed radiation can reemerge in the Fe K lines. The strong Fe line seen in the X-ray binary Hercules X-1 may be from a thick shell of photoionized plasma being held above the surface of the neutron star by the magnetic field (the so called Alfven shell; Basko 1980).

In all optically thick plasmas the energy absorbed from the continuum is emitted primarily as emission lines. However, many of the strongest lines are in the ultraviolet, such as the resonance lines of C IV λ1540 and O VI λ1035. These lines have energies below the 13.6 eV ionization threshold of H, and therefore can escape relatively easily from the plasma.

III.3.4 Fluorescence

Fluorescence is line emission by neutral or nearly neutral material illuminated by an external source. An example is the photosphere of the normal star in a binary system with an X-ray pulsar. Continuum from the pulsar is absorbed by the photosphere, and some of the energy reemerges as scattered or fluoresced X-radiation. The photosphere is dense enough that it remains at $T \geq 10^4$ K (typical densities are $\sim 10^{15}$ cm^{-3} so, for $L = 10^{37}$ erg s^{-1} and $D = 10^{11}$ cm, eq. 3.47 gives $\xi \sim 1$), so most elements are not highly ionized. The dominant opacity for the incident X-rays is K shell photoelectric absorption by the abundant elements of Table 1 with $Z \approx 6$, much as in the interstellar medium (see II.xxx). The absorbing ion is left with an inner K shell vacancy which is an autoionizing state (see III.2): it can either decay by the emission of an additional electron (the Auger process) or decay radiatively by emitting a K X-ray. The branching ratio for radiative decay is called the *fluorescent yield.* Because the probability for radiative transition increases strongly with Z (e.g. see eq. 3.7), so do the fluorescent yields. For C, N and O, the fluoresecent yields are < 0.5%, they are ~5% for Si and S but 35% for Fe. The opacity is lower for the Fe line, so it will be by far the strongest fluorescent line, assuming, of course, that the incident continuum contains photons with energies above the Fe K absorption threshold E_K(E_K = 7.1 keV for Fe I).

For solar abundances the opacity of neutral iron $\kappa_{Fe}(E)$ at energy $E \geq 7.1$ keV is

$$\kappa_{Fe}(E) = 1.6\times10^{-24}\, n_H\, A_{Fe}\, [E/7.1\ \mathrm{keV}]^3\ \mathrm{cm}^{-1}, \qquad 3.50$$

where A_{Fe} is the abundance of Fe relative to solar abundances (Table 1) and n_H is the density of hydrogen (Hatchett and Weaver 1977). At threshold, κ_{Fe} is comparable to the Compton opacity,

$$\kappa_C = \sigma_C n_e = 0.8\times10^{-24}\, n_H\ \mathrm{cm}^{-1}, \qquad 3.51$$

where we have used eq. II.xx and $n_e = 1.2n_H$, which assumes a He abundance of 10%.

Detailed radiative transfer calculations show that ~3% of the photons near E_K striking the atmosphere at normal incidence will reemerge in the Fe K line (Basko 1978). This *albedo* falls like ~ E^{-3} as one would expect from 3.50. Because κ_{Fe} and κ_C are comparable, ~30% of the Fe K photons will have been Compton scattered at least once.

III.4 DIAGNOSTICS OF CORONAL PLASMAS

The major goal of astronomical spectroscopy is the elucidation of the physical parameters of the emitting object. These include temperature or the distribution of temperatures, densities, elemental abundances, and state of evolution. Most diagnostic techniques have been developed in the study of the solar corona or of laboratory plasmas, both of which are thermal, although the methods could be readily adapted to photoionized plasmas.

III.4.1 Line Emission from Cosmic Sources

The measured strength of an X-ray emission line from a cosmic source is a complicated function of the physical parameters which one would like to determine. For coronal plasmas, the flux at Earth of a collisionally excited emission line of energy E is

$$F_i = [7.3\times10^{-6} e^{-\sigma_E N_H}/4\pi D^2]\,[n(X)/n(H)] \int \Omega'_i\, f_{X_r}(T)\, T^{-1/2}\, e^{-E/kT}\, \Xi(T)\, dT \text{ photons cm}^{-3}\text{ s}^{-1}, \quad 3.52$$

where $f_{X_r}(T) = [n(X^r)/n(X)]$, is the ionization fraction and

$$\Xi(T) = n_e^2(T)\, dV/dT \quad 3.53$$

is the differential volume emission measure, each at temperature T. Here we have used 3.34 and 3.16 for the emissivity of the i^{th} line of the r^{th} ionization stage of element X integrated over the emitting volume. The first exponential accounts for absorption by material along the line of sight with hydrogen column density N_H; σ_E is the appropriate absorption cross section at energy E (see II.xxx) and D is the distance to the source from Earth.

The integral in eq. 3.52 extends over the emitting volume. Even in the study of an isolated solar active region, the observed region will likely encompass material at different temperatures; it surely does for more distant stellar coronae, SNRs or clusters. In the case of a supernova remnant, one may also have regions of varying composition and ionization history. Depending on the quantity and quality of the data, one may attempt to model these effects as part of the analysis or be forced to average over them.

The standard spectroscopic technique is to use ratios of selected line strengths to isolate particular parameters. The ratio of any two lines is independent of D. For homogeneous plasma, ratios of lines from the same element are independent of abundance and those from the same ion are generally independent of ionization fraction (the exceptions are lines whose upper levels are populated by recombination or inner-shell ionization). The ratios of lines of nearly the same energy are nearly independent of N_H. More generally, N_H must be determined along with the other parameters of the source in order to relate lines in different parts of the spectrum unless the absorption is very small (as it generally is above ~1-2 keV or for nearby stars).

The spectra of Helium-like ions are particularly useful as diagnostics for temperature, density and ionization equilibrium. The principal spectral lines are the resonance (R), forbidden (F) and intercombination (I) lines shown in Figure 3.1. Satellite lines from Li-like species are also important as diagnostics of temperature, ionization disequilibrium, and departures from Maxwellian electron distributions (see Figure 3.9).

III.4.2 Temperature Diagnositics

The temperature distribution of plasma in sources for which statistical equilibrium can be established or plausibly assumed can be found by comparing the observed line strengths to those predicted by coronal models (see III.4.5). This applies to clusters of galaxies and stellar coronae (possibly excluding the early phases of stellar flares).

Assuming for the moment that D and N_H can be eliminated or are known, then Equation 3.52 can rewritten for the luminosity of the line:

$$L_i = 4\pi D^2 e^{\sigma_E N_H} F_i = \int [P_{lu}(T)/n_e^2]\, \Xi(T)\, dT, \qquad 3.54$$

where the factor in brackets is the emissivity of the line computed as a function of T in the coronal models (see eq. 3.34). If several F_i are measured, in principle it should be possible to invert 3.54 to find the distribution of emission measure with temperature, $\Xi(T)$. The temperature dependence of the line emissivity is dominated by the product $f_{X_r}(T)\, T^{-1/2}\, e^{-E/kT}$ (called the *contribution function* for the line). Typically the emissivity is significant over a temperature range of roughly a factor of 2-4, although the range is even larger for lines of H-like ions. This means that in practice it is difficult to invert 3.54, because the emissivity acts as a smoothing function which obscures the detailed shape of the emission measure distribution. Furthermore, even modest observational uncertainties in the values of L_i are amplified in the inversion process, giving large uncertainties in $\Xi(T)$.

Nevertheless, measurements of even a small number of lines that are prominent at different temperatures can define the overall shape of the emission measure distribution. If the relative abundances of the elements are uncertain, then one should use lines from different ionization stages of a single element, such as Fe. Even approximate temperature distributions can often be of great value in astrophysics, as in the establishment of cooling flows in clusters and galaxies.

With several lines of overlapping contribution functions, one can apply more refined methods. One approach is to parameterize Ξ as a function of T, such as a low order polynomial, and to fit the values of the parameters using the measured line ratios. Alternatively, one can convert the integral in equation 3.54 into a discrete sum and solve by iteration or by matrix inversion (preferably using smoothing and constraining Ξ to be non-negative).

In isolated regions of the solar corona or of a supernova remnant, it is possible that much of the emission comes from material that is nearly isothermal. For example, the emission measure distribution of a coronal loop is sharply peaked at a single temperature. In this case, selected line ratios can be used to make relatively precise measurements of the electron temperature T_e. For example, the ratio of two lines from the same ion depends only on two parameters T_e and N_H:

$$F_i/F_j = [\Omega_i/\Omega_j] \exp\{-(E_i-E_j)/kT_e - (\sigma_{E_i} - \sigma_{E_j})N_H\}. \qquad 3.55$$

A given measurement defines an allowed region in the parameter space of N_H vs. T_e. Additional measured ratios can be used to reduce the size of this region.

Examples of line ratios that can be used for this purpose are the Lyman α to Lyman β line ratio of H-like ions (see Fig. 3X), and the $1s^2\ {}^1S$ - $1s3p\ {}^1P$ to $1s^2\ {}^1S$ - $1s2p\ {}^1P$ line ratio of He-like ions. Equation 3.55 and fig 3.X show that such line ratios will only be useful diagnostics for temperatures around $(E_i - E_j)/k$. For plasma at much higher temperatures the ratio is insensitive to T_e and can only be used to set a lower limit.

The 3p-2s lines of ionized Fe are particularly useful as temperature diagnostics. Lines from every ionization stage between Fe XVII and Fe XXIV appear in the same region of the spectrum, between 0.7 and 1.2 keV. Their contribution functions peak at temperatures from $\sim 4\times10^6$ K to $\sim 2\times10^7$ K, so the relative strengths of these neighboring lines will trace the emission measure distribution from ~2 to 50×10^6 K. Within this range, specific line ratios can be used as temperature verniers. An example is the ratio of Fe XVIII $2p^5\ ^3P_{3/2}$ - $2p^4(^1D)3d\ ^2D_{5/2}$ plus $^2P_{3/2}$ (the line is a blend) at 873 eV (14.20 A) to Fe XVII $2p^6\ ^1S_0$ - $2p^53d\ ^1P_1$ at 826 eV (15.01 A). This ratio changes from 0.05 at 3×10^6 K to 0.65 at 5.5×10^6K (Rugge and McKenzie 1985). The ratio of a multiplet of $2p^6$ - $2p^54d$ lines of Fe XVII at 1011 and 1023 eV (12.12 and 12.26 A) to the 2p - 3d (826 eV) line of the same ion is sensitive to temperatures between 1.5×10^6 K and 10^7 K. Both these diagnostics require sufficient signal to noise and spectral resolution to measure lines that differ in intensity by factors up to twenty. Also, the contributions from unresolved satellite lines can be significant and must be included in the model spectra (Raymond and Smith 1986).

The ratio $G = (I+F)/R$ for He-like ions, where I, F and R are the intercombination, forbidden and resonance transitions (see Fig. 3.1), is sensitive to T_e, although it also measures departures from ionization equilibrium (see III.4.3; Gabriel 1972; Pradhan and Shull 1981). For plasma at equilibrium the G ratio is an effective diagnostic over approximately two octaves of temperature roughly centered at T_m, the temperature of peak resonance line emission. Typically, G varies by ± 50% over this range, with $G(T_m) \leq 1$. An element with atomic number Z has $T_m \sim 1.2\times10^4(Z-1)^{2.6}$ K (Pradhan and Shull 1981); values of T_m for some abundant elements are given in Table 3.3. Because the lines of the He-like ions are at nearly the same energy, the diagnostic is independent of N_H. On the other hand, at the densities of stellar coronae, the optical depth in the resonance line can become significant, which will artificially reduce G (see III.2A). In spectra with high signal-to-noise, a more sensitive temperature diagnostic is given by the ratio of the $1s^2\ ^1S$ - $1s3p\ ^1P$ line to the resonance line. This ratio varies by more than an order of magnitude over a decade of temperature, but it is generally less than ~0.1 (Keenan et al. 1987)

Satellite lines to the He-like triplet from dielectronic recombination to autoionizing states of the Li-like ion are also useful as temperature diagnostics. These lines are increasingly important for the elements above Si, because of the strong Z dependence of the branching ratio for radiative decay (see III.1.4). The strongest lines are from the multiplet $1s^22p\ ^2P$ - $1s2p^2\ ^2D$. Unfortunately, for elements with Z below ~20 these are closely blended with the forbidden line; even for Fe, resolving powers of ~1000 are required to separate them. When it can be measured, the intensity ratio of this multiplet of dielectronic satellites lines to the resonance line is very sensitive to temperature, and it is independent of ionization state. In Fe, the ratio is ~0.2 at T_m. It changes by nearly an order of magnitude for temperatures from $\sim 1/2\ T_m$ to $\sim 2\ T_m$, being stronger at lower temperatures as expected for a recombination line (see Bhalla, Gabriel and Presnyakov 1975, Doschek et al. 1980).

Line profile measurements, performed with spectrometers of resolving power > 1000, can in principle give a measure of the temperature of the emitting ions, T_i. In many sources this can be equated to T_e. For a given T_i, the dispersion of the line profile from thermal Doppler broadening is given by

$$\sigma_{los}/c = [(kT_i)/(Am_pc^2)]^{1/2} \quad 3.56$$

$$= 0.003\ (T_i/10^7K)^{1/2}\ (A/10)^{-1/2} \quad 3.57$$

where σ_{los} is the root-mean-square velocity dispersion along the line-of-sight and A is the atomic weight of the emitting ion. However, in most astronomical plasmas, the bulk velocities due to shocks, outflow, etc., will be comparable to or exceed the thermal velocities, making the temperature diagnostic ineffective. The spectra of solar flares exhibit complex and variable profiles which are attributed to bulk motions (Acton et al. 1981).

III.4.3 Diagnostics for Ionization Disequilibrium

Diagnostics for departures from ionization equilibrium (see III.2C) involve making independent determinations of T_e and of the relative populations of ionization states for a given ion. The ratio of the populations of neighboring states defines an ionization temperature T_z, the temperature at which that population ratio would occur in a coronal plasma at equilibrium. If $T_z \neq T_e$, then disequilibrium is indicated. If one can assume that the plasma was impulsive heated, as in a supernova remnant, then the degree of inequality of T_z and T_e is a measure of the ionization time $\tau = n_e t$. Generally, different ion pairs will give different values of T_z, but in a homogeneous plasma they should all imply the same τ.

In dealing with plasmas out of ionization equilibrium one must distinguish between lines formed primarily through collisional excitation from those that are primarily the by-product of recombination or ionization processes. The strengths of the former are proportional to the population of the emitting ion (eq. 3.52), whereas the latter depend on the relative populations of neighboring ionization states.

The ratio of the $1s3p$-$1s^2$ line from He-like ions to the Lyman α line of the H-like ion of the same element gives a very direct measure of the relative populations in these two stages and therefore of T_z. Both these lines are excited primarily by collisions. They have nearly the same energy, so their ratio is nearly independent of T_e and N_H, which can be separately determined as described above.

The ratio $G = (I+F)/R$ for He-like ions, which was described in section III.4.2 as a temperature diagnostic, is also sensitive to the relative populations of He-like and H-like ions and is therefore a measure of departures from ionization equilibrium (Gabriel and Jordan 1969, Mewe and Schrijver 1978, Pradhan 1983). This is because the ^3S and ^3P upper levels of the I and F lines can be fed by cascades following radiative recombination as well as by collisional excitation, whereas collisional excitation alone dominates the production of the R line. Cascades favor population of the triplet levels because of their higher statistical weight. In a plasma that is under-ionized relative to the equilibrium value for its electron temperature (i.e. $T_z < T_e$), the recombination rate is suppressed and G is smaller than its equilibrium value (Figure 3.10). Conversely, in a recombining, overionized plasma G will be larger. Another population process that operates to increase G in an ionizing plasma is inner shell ionization of Li-like ions. This also favors the triplet states because of their highter statistical weights. However, in most ionizing plasmas the Li-like state is relatively short lived (see Figure 3.6), so the recombination effect generally dominates. The value of G can also be affected by non-Maxwellian electron distributions (Section III.4.4).

Because G depends on both T_e and T_z, one or more additional diagnostics are required to establish each parameter independently. For example, T_e could be fixed using one of the diagnostics of section III.4.2 that is independent of ionization state, or T_z could be measured with the $1s3p$-$1s^2$ to Lyα ratio mentioned above. Data on G for several supernova remnants show evidence for departures from ionization equilibrium and give values of τ that are consistent with ages and densities deduced by other means (see Figure 3.7; Canizares 1989).

Another diagnostic for ionization disequilibrium makes use of the satellites to the He-like triplet. As noted in section III.4.2, the ratio of satellites from dielectronic recombination of He-like ions to the He-like resonance line is sensitive to T_e but independent of T_z, as both lines are proportional to the density of He-like ions alone. A second multiplet of satellite lines comes from the transitions $1s^22s\ ^2S$ - $1s2p(^1P)2s\ ^2P$. In addition to being fed by dielectronic recombination, the upper level can also be populated by inner shell excitation of Li-like ions. This is not true of the $1s^22p\ ^2P$ - $1s2p^2\ ^2D$ multiplet, whose upper level can only be obtained from the $1s^2\ ^2S$ Li-like ground state by a second order process. The ratio of the latter, dielectronic recombination satellites to the resonance line gives T_e, and the ratio of the inner-shell ionization satellites to R depends on the Li-like to He-like ratio, giving a measure of T_z. The diagnostic is only useful for high Z elements, such as Ca or Fe, which have sizable Li-like ionization fractions at temperatures around T_m. For example, for Fe the inner-shell satellite to resonance line ratio is ~0.06 at $T_z \sim T_m$, and is twice that for $T_z \sim 0.6\ T_m$. For O and even Si inner-shell ionization gives a negligible contribution to the ^2P upper level (Bhalla, Gabriel and Presnyakov 1975, Feldman, Doschek and Kreplin 1980, Bely-Dubau et al. 1982).

III.4.4 Diagnostics of the Electron Energy Distribution

Although the assumption of Maxwellian electron energy distributions has some justification in most astrophysical plasmas (section III.2.3) it would be comforting to have observational verification. Most collisionally excited lines are not very sensitive to the exact shape of the electron distribution (III.2.3; Owocki and Scudder 1983). Suprathermal electrons can affect the relative intensities of the He-like triplet, however, because very energetic electrons preferentially excite the singlet level. If as much as several percent of the electrons had energies ~10kT, they would affect the ratio G and compromise its utility as a diagnostic of T_e or T_z (Gabriel et al 1990). On the other hand, these energetic electrons would also radiate sufficient bremsstrahlung to reveal themselves.

Ratios of selected dielectronic satellites to He-like ions depend on the electron distribution. This is because the dielectronic recombination of an ion requires an electron of energy within a narrow range of the excitation energy of the intermediate autoionization state. For example, the $1s^22p\ ^2P_{3/2}$ - $1s2p^2\ ^2D_{5/2}$ line of Fe XXIV is formed by recombination of Fe XXV with a 4.694 keV electron. The $1s^23p\ ^2P_{3/2}$ - $1s2p3p(^1P)^2D_{5/2}$ line follows recombination of Fe XXV with a 5.185 keV electron. The ratio of the intensities of these lines to that of the resonance line, which is excited by any electron that exceeds the threshold energy of 6.7 keV, gives a measure of the population of electrons at 4.7 or 5.2 keV relative to those above 6.7 keV, which can be compared to that expected for a Maxwellian distribution. This method has indicated departures from an equilibrium electron energy distribution in the early minutes of solar flares (Gabriel and Phillips 1979, Seely, Feldman and Doschek 1987).

III4.5. Density Diagnostics

The ratio of the forbidden to the intercombination lines of He-like ions is a diagnostic of plasma density. The ratio increases when the rate of collisional excitation from the metastable $2\ ^3S$ state to the $2\ ^3P$ state becomes comparable to the $2\ ^3S$ to $2\ ^1S$ radiative decay rate (radiative excitation between triplet states is generally unimportant). The relation between the measured I/F ratio R_{IF} and n_e is

$$n_e = n_c(T)(R'_{IF}(T)/R_{IF} - 1),$$

where $n_c(T)$ is the critical density for the given element and $R'_{IF}(T)$ is value of R_{IF} in the low density limit. Both parameters are only weak functions of temperature, generally remaining within ~10% of their values at T_m. For O, Si and Fe the critical densities are 2×10^{10}, 4×10^{13} and 10^{17} cm^{-3} and R'_{IF} = 4, 2.5, and 1.0, respectively. (Pradhan 1982).

Generally it is possible to see density effects only if $n_e > n_c/10$, which is comparable to the densities in stellar coronae but is orders of magnitude above those in SNRs or clusters of galaxies. This diagnostic has been used to deduce the densities of solar active regions (McKenzie 1987, McKenzie and Landecker 1982) in the range 3-13x10^9 cm^{-3}.