

# Subject index

---

- Abelian weights, 143  
Abel's theorem, 147  
abscissa  
  of absolute convergence, 14  
  of convergence, 11  
arithmetic semigroup, 278  
Ayer's theorem, 247, 276
- Bernoulli numbers, 495ff  
Bernoulli polynomials, 495ff  
Bertrand's postulate, 49  
beta function, 530  
Beurling primes, 266ff, 277, 278, 483  
Blaschke product, 192  
Birch–Swinnerton-Dyer conjectures, 393  
Borel–Carathéodory lemma, 169  
Brun–Titchmarsh inequality, 90  
Buchstab's function, 216–220
- Catalan's constant, 514  
Catalan numbers, 8  
Cesàro summability, 147, 158  
Cesàro weights, 142  
character  
  additive, 108ff  
  Dirichlet, 115ff  
    complex, 123  
    conductor, 283  
    induced, 282  
    primitive, 282ff  
    quadratic, 295ff  
    real, 123  
  group, 133  
circle problem, 45–46  
covering congruences, 7
- critical line, 328  
critical strip, 328
- Dedekind zeta function, 194, 321, 343, 392  
Dickman function, 200, 201, 210–212  
differential–delay equation, 200, 216  
digamma function, 522ff  
Dirichlet character: *see* Character, Dirichlet  
Dirichlet convolution, 38  
Dirichlet divisor problem, 68  
Dirichlet–Jordan test, 542  
Dirichlet kernel, 535  
Dirichlet  $L$ -function, 120ff  
  analytic continuation, 121, 332–333  
  distribution of zeros, 351, 454–456  
  Euler product, 120, 121  
  exceptional zero, 360, 367ff  
  functional equation, 333  
  non-trivial zeros, 333, 358ff  
  special values, 337  
  trivial zeros, 333  
Dirichlet series, 1, 11ff, 137ff  
  formal, 39  
  generalized, 31  
  ordinary, 31  
Dirichlet's theorem  
  on Diophantine approximation, 478  
  on primes in a. p., 123  
discriminant, 343  
  quadratic, 296  
divisor function, 2, 38, 45–46, 55–56, 60, 68–69
- Euler numbers, 506  
Euler's constant, 26, 514

- Euler–Maclaurin summation formula, 25, 44, 500ff  
 Euler products, 19ff  
 Euler's totient function, 27, 36, 55, 68  
 explicit formulae, 397ff
- Farey fractions, 183, 184  
 finite differences, 510  
 finite Fourier transform, 109  
 Fourier series, 535ff  
 fractional part, 39  
 function,  
   additive, 21  
   arithmetic, 20  
   even, 133  
   multiplicative, 20  
   totally additive, 21  
   totally multiplicative, 20
- gamma function, 520ff  
 Artin's theorem, 520, 535  
 Euler's integral, 524, 532  
 Gauss's formula, 520, 531  
 Gauss's multiplication formula, 527, 532  
 Hankel's integral, 525  
 incomplete, 327  
 Legendre's duplication formula, 522, 532  
 Mellin's integral, 525, 529  
 reflection formula, 521, 532  
 special values of, 520ff  
 Stirling's formula, 523, 532  
 Weierstrass product, 520  
 Gauss sum, 286ff  
 generalized prime numbers,  
   *see* Beurling primes  
 Generalized Riemann Hypothesis, 333  
 generating function, 1  
 Größencharakter, 120, 132, 344, 366, 385  
 group representation, 133
- Hankel path, 515  
 Heisenberg uncertainty principle, 147  
 Hurwitz zeta function, 30, 340, 513
- inclusion–exclusion, 77  
 inversion formula,  
   Möbius, 35
- Jensen's formula, 168
- Kronecker symbol, 296  
 Kummer congruences, 514
- Lambert summability, 159  
 Landau's theorem, 16, 32, 463  
 lattice, 541  
 Lerch zeta function, 515  
 Lindelöf Hypothesis, 330, 438  
 Liouville lambda function, 21  
 logarithmic integral, 5, 180, 189ff
- von Mangoldt lambda function, 23  
 matrix,  
   unimodular, 541  
   unitary, 112, 119
- Mellin transform, 137, 141  
   inverse, 137, 141
- Mellin–Barnes integrals, 532  
 method of the hyperbola, 38  
 Mercer's theorem, 158  
 Minkowski's convex body theorem, 542  
 Möbius mu function, 21
- oscillation of error terms, 463ff
- Parseval's identity, 110, 133  
 partition, 7  
 Pell's equation, 134  
 Perron's formula, 137ff  
 Plancherel's identity, 144, 162  
 Poisson summation  
   formula, 538ff  
 Pólya–Vinogradov inequality, 307, 309, 322  
 power series, 1  
 power-full number, 66  
 Prime Ideal Theorem, 194, 267  
 Prime  $k$ -tuple conjecture, 103, 224  
 Prime Number Theorem, 3, 168ff, 244ff, 276, 277  
   elementary proof, 250ff  
   for arithmetic progressions, 358ff
- Ramanujan expansion, 133  
 Ramanujan sum, 110ff, 133, 265, 287  
 regular transformation, 148  
 Riemann Hypothesis, 328, 417  
   consequences of, 419ff  
   Generalized, 333  
 Riemann–Siegel formula, 515  
 Riemann–Roch theorem, 322  
 Riemann–Stieltjes integral, 12, 486ff  
   first mean value theorem for, 491  
   refinement, 492  
   second mean value theorem for, 492  
   uniform, 492

- Riemann zeta function, 2  
 analytic continuation, 24–27, 500, 501  
 distribution of zeros, 175, 353–354,  
     452ff  
 Euler product, 22  
 functional equation, 326ff  
 linear independence of zeros, 447ff,  
     467ff  
 non-trivial zero, 328  
 special values, 328  
 trivial zeros, 328  
 zero-free region, 168–175, 192–194  
 zeros on the critical line, 456ff  
 Riesz product, 482  
 Riesz representation theorem, 493  
 Riesz typical mean, 143  
 saw-tooth function, 536  
 secant coefficients, 506  
 sieve, 76ff  
     Brun, 78  
     combinatorial, 78  
     Eratosthenes–Legendre, 76  
     Selberg, 82ff, 102  
 sine integral, 139  
 square-free kernel, 84  
 square-free number, 36, 183, 186, 225, 446,  
     471  
 von Staudt–Clausen theorem, 512, 514  
 Stirling’s formula, 503  
 summability, 147–167  
     Abel, 147  
     Cesàro, 158  
     Lambert, 159  
     Riesz, 158  
 sums of two squares, 45, 46, 187, 188, 227,  
     228  
 symmetric group, 184  
 tangent coefficients, 505  
 Tauberian theorem, 150ff  
     Hardy–Littlewood, 151–155, 163  
     Hardy’s, 150  
     Karamata’s, 163  
     Littlewood’s, 151, 163  
     Tauber’s first, 150  
     Tauber’s second, 160–161  
     Wiener–Ikehara, 259–266, 277  
     Wiener’s, 163–164  
 Wallis’ formula, 503, 507  
 Weyl sum, 193