Notation

Symbol	Meaning	Found on page
$\mathbb{C}$	The set of complex numbers.	109
$\mathbb{F}_p$	A field of p elements.	9
$\mathbb{N}$	The set of natural numbers, 1, 2, ...	114
$\mathbb{Q}$	The set of rational numbers.	120
$\mathbb{R}$	The set of real numbers.	43
$\mathbb{T}$	$\mathbb{R}/\mathbb{Z}$, known as the *circle group* or the *one-dimensional torus*, which is to say the real numbers modulo 1.	110
$\mathbb{Z}$	The set of rational integers.	20
B	constant in the Hadamard product for $\xi(s)$	347, 349
B_k	Bernoulli numbers.	496ff
$B_k(x)$	Bernoulli polynomials.	45, 495ff
$B(\chi)$	constant in the Hadamard product for $\xi(s, \chi)$	351, 352
C_0	Euler's constant	26
$c_q(n)$	The sum of $e(an/q)$ with a running over a reduced residue system modulo q; known as *Ramanujan's sum*.	110
$c_\chi(n)$	$= \sum_{a=1}^{q} \chi(a)e(an/q)$.	286, 290
$d(n)$	The number of positive divisors of n, called the *divisor function*.	2
$d_k(n)$	The number of ordered k-tuples of positive integers whose product is n.	43
$E_0(\chi)$	$= 1$ if $\chi = \chi_0$, 0 otherwise.	358

Symbol	Meaning	Found on page
E_k	The *Euler numbers*, also known as the *secant coefficients*.	506
$e(\theta)$	$= e^{2\pi i\theta}$; the complex exponential with period 1.	64, 108ff
$L(s,\chi)$	A Dirichlet L-function.	120
$\mathrm{Li}(x)$	$= \int_0^x \frac{du}{\log u}$ with the Cauchy principal value taken at 1; the *logarithmic integral*.	189
$\mathrm{li}(x)$	$= \int_2^x \frac{du}{\log u}$; the *logarithmic integral*.	5
$M(x)$	$= \sum_{n\le x} \mu(n)$	182
$M(x;q,a)$	The sum of $\mu(n)$ over those $n \le x$ for which $n \equiv a \pmod q$.	383
$M(x,\chi)$	The sum of $\chi(n)\mu(n)$ over those $n \le x$.	383
$N(T)$	The number of zeros $\rho = \beta + i\gamma$ of $\zeta(s)$ with $0 < \gamma \le T$.	348, 452ff
$N(T,\chi)$	The number of zeros $\rho = \beta + i\gamma$ of $L(s,\chi)$ with $\beta > 0$ and $0 \le \beta \le T$.	454
$P(n)$	The largest prime factor of n.	202
$Q(x)$	the number of square-free numbers not exceeding x	36
$S(t)$	$= \frac{1}{\pi}\arg\zeta(\frac{1}{2}+it)$.	452
$S(t,\chi)$	$= \frac{1}{\pi}\arg L(\frac{1}{2}+it,\chi)$.	454
$\mathrm{si}(x)$	$= -\int_x^\infty \frac{\sin u}{u}\,du$; the *sine integral*.	139
T_k	The *tangent coefficients*.	505
$w(u)$	The *Buchstab function*, defined by the equation $(uw(u))' = w(u-1)$ for $u > 2$ together with the initial condition $w(u) = 1/u$ for $1 < u \le 2$.	216
$Z(t)$	Hardy's function. The function $Z(t)$ is real-valued, and $\lvert Z(t)\rvert = \lvert\zeta(\frac{1}{2}+it)\rvert$.	456ff
β	The real part of a zero of the zeta function or of an L-function.	173
$\Gamma(s)$	$= \int_0^\infty e^{-x}x^{s-1}\,dx$ for $\sigma > 0$; called the *Gamma function*.	30, 520ff

Symbol	Meaning	Found on page
$\Gamma(s,a)$	$=\int_a^\infty e^{-w}w^{s-1}\,dw$; the *incomplete Gamma function*.	327
γ	The imaginary part of a zero of the zeta function or of an L-function.	172
$\Delta_N(\theta)$	$=1+2\sum_{n=1}^{N-1}(1-n/N)\cos 2\pi n\theta$; known as the *Fejér kernel*.	174
$\varepsilon(\chi)$	$=\tau(\chi)/\big(i^\kappa q^{1/2}\big)$.	332
$\zeta(s)$	$=\sum_{n=1}^\infty n^{-s}$ for $\sigma>1$, known as *the Riemann zeta function*.	2
$\zeta(s,\alpha)$	$=\sum_{n=0}^\infty (n+\alpha)^{-s}$ for $\sigma>1$; known as the *Hurwitz zeta function*.	30
$\zeta_K(s)$	$\sum_{\mathfrak{a}} N(\mathfrak{a})^{-s}$; known as the *Dedekind zeta function* of the algebraic number field K.	343
Θ	$=\sup \Re\rho$	430, 463
$\vartheta(x)$	$=\sum_{p\le x}\log p$.	46
$\vartheta(z)$	$=\sum_{n=-\infty}^\infty e^{-\pi n^2 z}$ for $\Re z>0$.	329
$\vartheta(x;q,a)$	The sum of $\log p$ over primes $p\le x$ for which $p\equiv a \pmod q$.	128, 377ff
$\vartheta(x,\chi)$	$=\sum_{p\le x}\chi(p)\log p$.	377ff
κ	$=(1-\chi(-1))/2$.	332
$\Lambda(n)$	$=\log p$ if $n=p^k$, $=0$ otherwise; known as the *von Mangoldt Lambda function*.	23
$\Lambda_2(n)$	$=\Lambda(n)\log n+\sum_{bc=n}\Lambda(b)\Lambda(c)$.	251
$\Lambda(x;q,a)$	The sum of $\lambda(n)$ over those $n\le x$ such that $n\equiv a \pmod q$.	383
$\Lambda(x,\chi)$	$=\sum_{n\le x}\chi(n)\lambda(n)$.	383
$\lambda(n)$	$=(-1)^{\Omega(n)}$; known as the *Liouville lambda function*.	21
$\mu(n)$	$=(-1)^{\omega(n)}$ for square-free n, $=0$ otherwise. Known as the *Möbius mu function*.	21
$\mu(\sigma)$	the Lindelöf mu function	330
$\xi(s)$	$=\frac{1}{2}s(s-1)\zeta(s)\Gamma(s/2)\pi^{-s/2}$.	328
$\xi(s,\chi)$	$=L(s,\chi)\Gamma((s+\kappa)/2)(q/\pi)^{(s+\kappa)/2}$ where χ is a primitive character modulo q, $q>1$.	333

Symbol	Meaning	Found on page
$\Pi(x)$	$= \sum_{n\le x} \Lambda(n)/\log n$.	416
$\pi(x)$	The number of primes not exceeding x.	3
$\pi(x;q,a)$	The number of $p \le x$ such that $p \equiv a \pmod q$,.	90, 358
$\pi(x,\chi)$	$= \sum_{p\le x} \chi(p)$.	377ff
ρ	$= \beta + i\gamma$; a zero of the zeta function or of an L-function.	173
$\rho(u)$	The *Dickman function*, defined by the equation $u\rho'(u) = -\rho(u-1)$ for $u > 1$ together with the initial condition $\rho(u) = 1$ for $0 \le u \le 1$.	200
$\sigma(n)$	The sum of the positive divisors of n.	27
$\sigma_a(n)$	$= \sum_{d\mid n} d^a$.	28
τ	$= \|t\| + 4$.	14
$\tau(\chi)$	$= \sum_{a=1}^{q} \chi(a)e(a/q)$; known as the *Gauss sum* of χ.	286ff
$\Phi_q(z)$	The q^{th} cyclotomic polynomial, which is to say a monic polynomial with integral coefficients, of degree $\varphi(q)$, whose roots are the numbers $e(a/q)$ for $(a,q) = 1$.	64
$\Phi(x,y)$	The number of $n \le x$ such that all prime factors of n are $\ge y$.	215
$\Phi(y)$	$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-t^2/2}\,dt$; the cumulative distribution function of a normal random variable with mean 0 and variance 1.	235
$\varphi(n)$	The number of a, $1 \le a \le n$, for which $(a,n) = 1$; known as *Euler's totient function*.	27
$\chi(n)$	A Dirichlet character.	115
$\psi(x)$	$= \sum_{n\le x} \Lambda(n)$.	46
$\psi(x,y)$	The number of $n \le x$ composed entirely of primes $p \le y$.	199
$\psi(x;q,a)$	The sum of $\Lambda(n)$ over $n \le x$ for which $n \equiv a \pmod q$.	128, 377ff
$\psi(x,\chi)$	$= \sum_{n\le x} \chi(n)\Lambda(n)$.	377ff
$\Omega(n)$	The number of prime factors of n, counting multiplicity.	21
$\omega(n)$	The number of distinct primes dividing n.	21

Symbol	Meaning	Found on page
$[x]$	The unique integer such that $[x] \leq x < [x] + 1$; called the *integer part* of x.	15, 24
$\{x\}$	$= x - [x]$; called the *fractional part* of x.	24
$\|x\|$	The distance from x to the nearest integer.	477
$f(x) = O(g(x))$	$\lvert f(x)\rvert \leq Cg(x)$ where C is an absolute constant.	3
$f(x) = o(g(x))$	$\lim f(x)/g(x) = 0$.	3
$f(x) \ll g(x)$	$f(x) = O(g(x))$.	3
$f(x) \gg g(x)$	$g(x) = O(f(x))$, g non-negative.	4
$f(x) \asymp g(x)$	$cf(x) \leq g(x) \leq Cf(x)$ for some positive absolute constants c, C.	4
$f(x) \sim g(x)$	$\lim f(x)/g(x) = 1$.	3