Statements made by Dr. Vaughan during
Math 568: Analytic Number Theory
Spring 2012
The Mathematics Department
The Pennsylvania State University

1. "Anything to do with primes tends to be awkward."

2. "Analytic Number Theory is full of world records. Some are more interesting than others."

3. "Now where were we? OH yes! I stated a theorem and now I need to prove it. There are more interesting things to do than prove theorems. It's trivial anyway."

4. "Modern Complex Analysis was developed to prove the Prime Number Theorem."

5. "Why does one climb a mountain? Because it's there. Why does one calculate a constant? Because it's there."

6. "[The Möbius Function] is a remarkable function ...and I don't have any time to tell you about it today."

7. Jake: Why do we have two Theorem 1.12's?
   Dr. Vaughan: Because I can't count. You should see me trying to calculate a tip at a restaurant.
   Jake: You just divide by 6.
   Dr. Vaughan: But that's too hard to do!
   Jake: But you're a number theorist!
   Dr. Vaughan: That's precisely correct. I know the *theory* of dividing by 6.

8. "I'm surprised you go to class at all. I didn't go to class when I was a grad student. *pause* I probably shouldn't have said that."

9. "What I want to do, I don't have time to do because I have been waffling so much."

10. "I had a different function here, but I think I'll do something else that I thought about last night. So I'm scrapping my notes."

11. "What's the use of averages? Sometimes nothing more than an exercise."

12. (After starting a sum at $n = 2$ instead of $n = 1$) "This is because $\log\log 1$ is too interesting of a number."

13. "The proof isn't a big deal."

14. "Statisticians would throw up their arms, but that's their problem."

15. "As a general rule, $\frac{1}{2\pi i}$ is well known to equal 1."

16. "That's just a momentary slip on my part. I had too good of a lunch."

17. "I need to do my usual trickery on partial sums."

18. "By 'totally multiplicative,' I mean multiplicative as the man on the street would understand."

19. "Analytic Number Theorists have about three tricks: (1) Cauchy-Schwartz Inequality, (2) Express your quantity as a double sum and interchange the order, and (3) Hölder's Inequality."

20. "And this is the best you can do without working very hard, and I don't want to work us very hard...contrary to what you might think."

21. "That begs an interesting question. Which came first: the zeros or the primes?"

22. "I'm going to leave out the annoying even prime. It's a bit like left handers in baseball: annoying to deal with."

23. "So, I mean any sporting personality...Tiger Woods might have an Erdős number of 3!"

24. "I know a story about ... I should talk about mathematics, shouldn't I?"

25. "If... and this is a big if, or a fairly big if."

26. "All sensible functions in my mind have period 1, not $2\pi i$."

27. "It always takes a long time for me to say things. I don't know why."

28. "Computer scientists have come late to the party..."

29. "I don't know why it's Theorem 1. *pause* Oh, I suppose it's the first theorem."

30. Most popular phrase (other than *prime number theorem, Dirichlet Series, Riemann-Zeta...*): Plan of campaign.

31. "I only know a few anecdotes [about the history of mathematics], really."

32. "I think it's nice using number theory to evaluate an integral."

33. "Think of a theorem like a sporting event. Is he going to get the proof? Is he?"

34. "I like the proof here. It's so profound. The first proof you see is combinatorial and looks like an accident. This one doesn't look like an accident."

35. "Let me begin by summarizing everything we know."

36. "Poisson didn't know how to prove *Poisson Summation Formula*. But we won't worry about that."

37. "The *Poisson Summation Formula*...perhaps it really does contain the secrets of the universe. Paul Cohen used to think so."

38. "I'm going to assume that your know everything there is to know about the Gamma Function."

39. "Whatever your favorite convergence theorem is...any of those will justify this interchange."

40. Dr. Vaughan: "How much does a pint of Guinness cost in Dublin?"
    Ayla: "You could go to the Guinness factory and get one for free."
    Dr. Vaughan: "Then that makes Dublin the cheapest [place in Europe]."

41. "These days, if people want to solve a Laplace transform, they sit down at a computer and it spits out an answer. It's sad really."

42. "This 2 may not be quite 2."

43. "Oh that only has three $\log' s$ in it. That's disappointing."

44. **"Desperate people prove desperate theorems."**

45. "... 'centered'... that's the American spelling. I must be infected."

46. "Actually, I have just spent half an hour doing nothing."

47. "This argument is for my convenience, not for calculating constants."

48. "Now we have to put up with ungrammatical mathematics."

49. Dr. Vaughan presented Rouché's Theorem and said, "There it's a beautiful theorem: Rouché's Theorem. I'm not going to use it, I just think it's beautiful."

50. "We'll consult the oracle." [Pulls out his book]

51. "It's actually amazing at how unsuccessful this machine is. About two-hundred years of mathematicians' work went into this."

52. "So what are we doing in this class? Well, that's a very good question."

53. "Complex Analysis is the most beautiful field in mathematics. And quite possibly the most beautiful human endeavor."

54. "I think these blackboards are intended for someone six inches shorter than I am."

55. "There is something fishy going on here. I shall just follow my notes."

56. "Cauchy had so many theorems named after him. He sent an average of three papers per week off. He probably had nothing better to do."

57. "And the ϕ here? It looks like another wrinkle, like something a professor just puts there."

58. "If you're not careful, you can lose out by a factor of 2. But who cares? We save it because we can."

59. "If you adopt this, you can forget about this term since it becomes a constant and you can add it to that other constant."

60. "Such a beautiful piece of chalk, I don't want to waste it by breaking it. Oh. There are three more of them. Then it's ok."

61. "What I am going to do on Thursday is prove this lemma, and then perhaps we can say something about the ζ function."

62. "I am sorry I spent a little bit of time doing analysis instead of Number Theory."

63. "One of the most powerful machines in mathematics is positivity."

64. "The whole point of the inequality is because 4 is greater than 3."

65. "There are quite a number of theorems which would be easy to prove if you didn't have to worry about conditional convergence."

66. "Estermann has a constant of 1 over 20, or something. I think he can do better than that; he wasn't working that hard."

67. "It doesn't make sense to look at primes if $x < 2$."

68. "Möbius hadn't invented his function, yet, but Riemann wrote down the formula anyway."

69. "I should just continue to talk about the Prime Number Theorem."

70. "...then you can forget about everything."

71. "It's something you either get used to, or you don't. If you get used to it, you become an analytic number theorist. If you don't, then you become an algebraic number theorist."

72. "You get some transcendental equation and can spin your wheels for awhile on that."

73. "If you want the most precise result that's ever been proved, you can throw in a whole bunch of silly terms."

74. "Sometimes, it seems like magic if you don't know what is going on."

75. "The world of Analytic Number Theory is divided into two camps, and neither can agree because they can't agree on terminology."

76. "Why the word 'smooth' came into fashion, I don't know."

77. "In this world, you're more likely to get funded if you use buzzwords...I'm going to use 'smooth numbers' just sounds so much better."

78. "This is very bad notation and I don't know whom we have to thank for it. Probably Dickman, whoever he was."

79. "So I've just proved $0 > 0$." [Jake claps] "Thank you."

80. "The upper one of these inequalities is actually closer to the truth. It's actually sort of interesting what you can get away with."

81. "Again, that article was more like a book; it was around a hundred pages."

82. "This is a good place to stop...just as you get to the interesting part."

83. "Strictly speaking, what I've written here is wrong."

84. "I'll need to understand what's going on in this proof at some stage."

85. "Chalk is a wonderful medium. In the days when you had to write on paper and pen, you couldn't correct things this easily."

86. "This is basic calculus. You can probably do it a lot better than I can."

87. **"Sometimes I feel like Fermat: this blackboard is not wide enough to contain the proof."**

88. "There is one further thing I wanted to say, but I won't prove anything."

89. "Apparently [Bombieri] was an expert on Tasmanian stamps from the 1920's."

90. "And you might ask, 'what did Walfisz do?' Many of us think nothing, except to get his name on the theorem."

91. "Landau may have been the first to write it down, but it's so trivial that I doubt anyone would want their name attached to it."

92. "And after all, we're dealing with something that probably doesn't exist at all."

93. "I should look at my notes, but I'm being pig-headed, here."

94. "Never once did Siegel mention zeros in his paper; he mentioned the class number. That was the 'Holy Grail' at the time."

95. "Estermann's result is the one I am going to give you. It's short, it's slick, and no one knows why it works."

96. "Now the question you should ask yourself is: 'why on earth $\frac{19}{20}$?' ... Because it's special."

97. "That's the worst calculation you'll see in this class."

98. "Is this actually mathematics?"

99. "I have a theorem that we shouldn't have any fall semester before Thanksgiving break."

100. "We've already defined constants in terms of things we don't know."

101. "So far, we haven't proved anything really. Just established some definitions, which are probably vacuous anyway."

102. "That's why I became a professor; I wanted to waste my whole life."

103. "In America, if someone's a lousy professor, you want to make them a professor as soon as possible so they're only teaching grad students."

104. "Basically, I hand wave and it's obvious that the coefficients are non-negative."

105. " 'Clearly' – It's a good word to use when it's complicated and you don't want to explain it."

106. "Pronouncing Dutch names is worse than pronouncing anything else."

107. "Well, I think that we've finished with multiplicative number theory."

108. "Unfortunately, things are not quite as rosy."

109. "One of the world's leading authorities on this is Dale Brownawell. I think you should pick his brain."

110. "So, of course, Dirichlet proved the best theorem."

111. Jake: "Is the homework hard?"
Dr. Vaughan: "No, I think it's trivial."

112. "Everything we know is trivial, and the rest is a Field's Medal. There is nothing between."