MATH 571 ANALYTIC NUMBER THEORY II SPRING 2025, SYLLABUS

Class number: 26201, **Class ID**: 029708
Instructor: Bob Vaughan, 335 McAllister
Email: rcv4psu.edu
Web: `https://personal.science.psu.edu/rcv4/`
Office Hours: by arrangement.
Class: 3:35-4:25pm MWF 113 McAllister.
Text: Class Notes "`https://personal.science.psu.edu/rcv4/571s25/571s25.html` or `https://personal.science.psu.edu/rcv4/Vol2/Vol2.pdf`
• The following give useful background:
H. L. Montgomery & R. C. Vaughan, Multiplicative Number Theory I. Classical Theory, Cambridge University Press, xii + 516pp, 2006.
Harold Davenport, Multiplicative Number Theory, third edition revised by Hugh Montgomery, Springer-Verlag, 2000.
Gérald Tenenbaun, Introduction to Analytic and Probabilistic Number Theory, Cambridge University Press, 1995, ISBN 0521412617.
R. C. Vaughan, The Hardy-Littlewood Method, second edition, Cambridge University Press, 1997.
•Homework: Due every Monday, except when Monday is a holiday, when due Wednesday.
•Grading: Based on Homework.

Topics

There have been several recent sensational developments in analytic number theory.

1. Zhang, Maynard and Tao, following Goldston, Pintz and Yildirim showed that there are bounded gaps between primes.
2. Green and Tao have shown that there are arbitrarily long arithmetic progressions in the primes.
3. Helfgott has shown that every odd $n > 5$ is the sum of three prime numbers.
4. Matomäki and Radziwill have proved interesting new results about multiplicative function.
5. There have been breakthroughs on the Vinogradov Mean Value Theorem by Wooley, Bourgain and others.
6. Koukoulopoulos and Maynard proved the Duffin and Schaeffer conjecture.
7. Guth and Maynard have broken the $\frac{3}{4}$ barrier on zero density esimates. This course is largely self-contained and will cover the large and Selberg sieves, Bombieri's theorem on primes in a.p., which says that the Generalised Riemann Hypothesis is true on average, and the Zhang-Maynard-Tao theorems on bounded prime gaps. Otherwise I am open to suggestions. Any of the above are possible. We could also discuss work on the distribution of the zeros of the Riemann zeta function or some of the fairly recent work on Waring's problem.

• Late homework will not be accepted unless prior permission is granted. No homework will be accepted after the graded ones have been returned to the students.
• All Penn State Policies regarding academic integrity apply to this course. Academic integrity is the pursuit of scholarly activity in an open, honest and responsible manner. Academic integrity is a basic guiding principle for all academic activity at The Pennsylvania State University, and all members of the University community are expected to act in accordance with this principle. Consistent with this expectation, the University's Code of Conduct states that all students should act with personal integrity, respect other students' dignity, rights and property, and help create and maintain an environment in which all can succeed through the fruits of their efforts.

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• Penn State welcomes students with disabilities into the University's educational programs. Every Penn State campus has an office for students with disabilities. Student Disability Resources (SDR) website provides contact information for every Penn State campus

`https://equity.psu.edu/offices/student-disability-resources`

For further information, please visit Student Disability Resources website

`http://equity.psu.edu/sdr/`

In order to receive consideration for reasonable accommodations, you must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: See documentation guidelines `http://equity.psu.edu/sdr/guidelines` If the documentation supports your request for reasonable accommodations, your campus disability services office will provide you with an accommodation letter. Please share this letter with your instructors and discuss the accommodations with them as early as possible. You must follow this process for every semester that you request accommodations.

• Many students at Penn State face personal challenges or have psychological needs that may interfere with their academic progress, social development, or emotional wellbeing. The university offers a variety of confidential services to help you through difficult times, including individual and group counseling, crisis intervention, consultations, online chats, and mental health screenings. These services are provided by staff who welcome all students and embrace a philosophy respectful of clients' cultural and religious backgrounds, and sensitive to differences in race, ability, gender identity and sexual orientation.
Counseling and Psychological Services at University Park (CAPS)
(`http://studentaffairs.psu.edu/counseling/`): 814-863-0395
Counseling and Psychological Services at Commonwealth Campuses
(`https://studentaffairs.psu.edu/counseling/caps-campuses`)
Penn State Crisis Line (24 hours/7 days/week): 877-229-6400. Crisis Text Line (24 hours/7 days/week): Text LIONS to 741741

• Consistent with University Policy AD29, students who believe they have experienced or observed a hate crime, an act of intolerance, discrimination, or harassment that occurs at Penn State are urged to report these incidents as outlined on the University's Report Bias webpage (`http://equity.psu.edu/reportbias/`)

• When searching for help with your course, your instructor will always be the best resource for advice. The University is aware of fraudulent companies and scammers who prey on a student's desire for assistance with coursework. A student who hires someone to do work on their behalf demonstrates a clear example of a violation of academic integrity highlighted above. Sharing login credentials with others is also in violation of Penn State Policy AD95 and may be subject to additional disciplinary action. Furthermore, a fraudulent company and/or scammer may extort the student by threats to communicate the student's arrangement with the University. If you find yourself in such a situation, it is best to contact your instructor immediately.