MATH 571, SPRING 2023, PROBLEMS 11

Due Monday 3rd April

In a letter dated 7 June 1742, from Moscow, to Euler, as part of their discussion about sums of primes and what are now known as the Goldbach conjectures, Goldbach states another conjecture, namely that "Every integer which can be written as the sum of two primes, can also be written as the sum of as many primes as one wishes, until all terms are units". Of course he included 1 as a prime.

1. Assume that every odd $n \geq 7$ is the sum of three primes. Prove that for every odd $n \geq 7$ and every k with $3 \leq k \leq (n-1)/2$, n is the sum of k primes.

2. Assume that every even $n \geq 4$ is the sum of two primes. Prove that for every even $n \geq 4$ and every k with $2 \leq k \leq n/2$, n is the sum of k primes.

3. Include 1 as a prime and assume every $n \geq 3$ is the sum of three primes. Prove Goldbach's above conjecture.