MATH 504 ANALYSIS IN EUCLIDEAN SPACES, SPRING TERM 2009, PROBLEMS 10

Return by Monday 6th April

1. This is based on the material in §2.7.1. Suppose that $f \in S \cap L^1(\mathbb{R})$ is given and $u \in C^2(\mathbb{R}) \cap L^1(\mathbb{R})$ satisfies $u'' - u = -f$.

(i) Show that $\lim_{|x|\infty} u(x) = \lim_{|x|\infty} u'(x) = 0$. Hint; the summability of u and f are useful here.

(ii) Show that $\widehat{(u'')}$ exists and equals $-4\pi^2 t^2 \hat{u}$. Hint; consider integration by parts.

(iii) Show that $(1 + 4\pi^2 t^2)\hat{u} = \hat{f}$.

(iv) Deduce that $u = \big((1+4\pi^2 t^2)^{-1}\hat{f}\big)^{\vee} = \frac{1}{2}e^{-|x|} \circ f = \frac{1}{2}\int_{\mathbb{R}} e^{-|x-y|} f(y)dx$. Hint; homework 9, question 2 is useful here.