# MATH 504 ANALYSIS IN EUCLIDEAN SPACES, SPRING TERM 2009, PROBLEMS 9

*Return by Monday 30th March*

1. (Exercise 2.6.5) Prove that if $\hat{f} \in L^1(\mathbb{R})$ then

$$\lim_{u \to 0+} \left( \exp(-2\pi^2 t^2 u)\hat{f}(t) \right)^{\check{}} = f(x)$$

pointwise.

2. (Exercise 2.6.6) Prove that if $u > 0$ and

$$k_u(x) = \frac{u}{\pi(u^2 + x^2)},$$

then

$$\hat{h}_u(t) = \exp(-2\pi u|t|).$$

Hint: The inverse transform is an elementary integral.

3. (Exercise 2.6.7) Use the previous exercise to prove that

$$\lim_{u \to 0+} \| (\exp(-2\pi^2 u|t|)\hat{f}(t))^{\check{}} - f(x) \|_1 = 0.$$