MATH 504 ANALYSIS IN EUCLIDEAN SPACES, SPRING TERM 2009, PROBLEMS 7

Return by Monday 16th March

1. Prove that if f, $g \in L^1(\mathbb{R})$, then $\|f \circ g\|_1 \le \|f\|_1 \|g\|_1$.

2. Let $X > 0$ and define

$$f(x) = \begin{cases} \frac{1}{X}\left(\frac{\sin \pi X x}{\pi x}\right)^2 & (x \neq 0), \\ X & (x = 0). \end{cases}$$

Prove that

$$\hat{f}(t) = \max\left(0, 1 - \frac{|t|}{X}\right)$$

and that the inverse Fourier transform of $\hat{f}$ is f. This is a very useful Fourier transform as it can be used to pick out expressions E with the property $|E| \le X$ yet the transform converges nicely and is non–negative.

3. Prove that (i) $\exp(-x^2)$ belongs to the Schwarz class but that (ii) $\frac{1}{1+x^2}$ and (iii) $\exp(-|x|)$ do not.