

**MATH 504 ANALYSIS IN EUCLIDEAN
SPACES, SPRING TERM 2009, PROBLEMS 4**

Return by Monday 9th February

1. §1.5 Exercise 1. Show that there are α, β in $L^1(S^1)$ such that $\|\alpha + \beta\|_1^2 + \|\alpha - \beta\|_1^2 \neq 2\|\alpha\|_1^2 + 2\|\beta\|_1^2$.
2. §1.5 Exercise 8. Prove that if $f \in L^1(S^1)$ and if for each fixed $x \in S^1$ the function $g_x(y) = y^{-1}(f(x+y) - f(x))$ is also in $L^1(S^1)$, then $S_n(x) = \sum_{|k| \leq n} \hat{f}(k)e_k(x)$ converges to f . Hint: Use the Dirichlet kernel and the Riemann–Lebesgue lemma.
3. §1.5 Exercise 12. Prove that the operation of convolution on $L^1(S^1)$ does not have an identity element. Hint: The Riemann–Lebesgue Lemma is useful here.