

**MATH 504 ANALYSIS IN EUCLIDEAN
SPACES, SPRING TERM 2009, PROBLEMS 2**

Return by Monday 26th January

These exercises are essentially the same as in the text, so I have included a cross reference.

1. §1.4. Exercise 5. Prove that if $\{z_n\}$ is a sequence of complex numbers such that $\lim_{n \rightarrow \infty} z_n = z$, then $\lim_{n \rightarrow \infty} \frac{1}{n}(z_1 + \cdots + z_n) = z$. Give an example in which the second limit exists but the first does not.

2. §1.4. Exercise 8. Suppose that $f \in \mathcal{C}(S^1)$ and $0 \leq r < 1$. Prove that

$$\sum_n \hat{f}(n) r^{|n|} e(nx) = \int_0^1 \frac{1 - r^2}{1 - 2r \cos(2\pi(x - y)) + r^2} f(y) dy.$$

3. §1.5. Exercises 4 and 5. Find a function f in $L^1[0, 1]$ which is not in $L^2[0, 1]$, and a function g in $L^2(\mathbb{R})$ which is not in $L^1(\mathbb{R})$.