MATH 467 FACTORIZATION AND PRIMALITY TESTING, FALL 2025, PROBLEMS 6

Return by Monday 6th October

1. 2. Solve where possible.
(i) $91x \equiv 84 \pmod{143}$
(ii) $91x \equiv 84 \pmod{147}$

2. Suppose that $m_1, m_2 \in \mathbb{N}$, $(m_1, m_2) = 1$, $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$ if and only if $a \equiv b \pmod{m_1 m_2}$.

3. Given that n is a product of two primes p and q with $p < q$, prove that

$$p = \frac{n + 1 - \phi(n) - \sqrt{(n + 1 - \phi(n))^2 - 4n}}{2}.$$

When $n = 19749361535894833$ and $\phi(n) = 19749361232517120$ use this to find p and q.

4. Solve the simultaneous congruences

$$\begin{aligned} x &\equiv 3 \pmod{6} \\ x &\equiv 5 \pmod{35} \\ x &\equiv 7 \pmod{143} \\ x &\equiv 11 \pmod{323} \end{aligned}$$

5. Eggs in basket problem (India 7c.). Find the smallest number of eggs such that when eggs are removed 2, 3, 4, 5 or 6 at a time 1 remains, but when eggs are removed 7 at a time none remain.

6. Find all solutions (if there are any) to each of the following congruences
(i) $x^2 \equiv -1 \pmod{7}$,
(ii) $x^2 \equiv -1 \pmod{13}$,
(iii) $x^5 + 4x \equiv 0 \pmod{5}$.