MATH 467 FACTORIZATION AND PRIMALITY TESTING, FALL 2025, PROBLEMS 3

Return by Monday 15th September

1. Find the complete solution in integers x, y to

$$16802x + 2015y = (16802, 2015)$$

2. Write a program to find x and y such that $mx + ny = \gcd(m, n)$ where
(i) $m = 8148657527$, $n = 8148653735$,
(ii) $m = 8418785375$, $n = 7849911069$,
(iii) $m = 4029583209458450398503$, $n = 3449459408504500003009$,
(iv) $m = 304250263527210$, $n = 230958203482321$.
A copy of your program should be submitted with your solutions to gain credit.

3. Let $\{F_n : n = 0, 1, \dots\}$ be the Fibonacci sequence as defined in Question 5 on Problem Sheet 2. Suppose that a and b are positive integers with $b \le a$ and we adopt the notation used in the description of Euclid's algorithm. Prove that for $k = 0, 1, \dots, s-1$ we have $F_k \le r_{s-1-k}$ and

$$s \le 1 + \frac{\log 2b\sqrt{5}}{\log \theta}.$$

This shows that Euclid's algorithm runs in time at most linear in the bit size of $\min(a, b)$.

4. The squarefree numbers are the natural numbers which have no repeated prime factors, e.g 6, 105. Note that 1 is the only natural number which is both squarefree and a perfect square. Prove that every $n \in \mathbb{N}$ with $n > 1$ can be written uniquely as the product of a perfect square and a squarefree number.