

Math 467 Factorization & Primality Testing, Fall 2025, Final Project

The task is to program the quadratic sieve as described in the QS handout with the theoretical choice for B for the size of the factor base, and to apply the program to the numbers n below. The project is divided into two parts. If you submit the first half on Canvas by 5th December I will give you feedback as to progress. **The final version of the project is due Monday 15th December.**

Part I: The sieving

For each of the numbers n below do the following.

1. List the primes in the factor base and the number K of primes in the factor base, including 2 (−1 is also in the factor base but is not prime).
2. List $K + 2$ values of x for which $x^2 - n$ completely factors over the factor base (here K is the number of primes in the factor base, including 2).
3. For each x in 2. give the factorization of $x^2 - n$. A vector of exponents suffices.

Part II: The linear algebra and factorisation

The task is to complete programming the quadratic sieve as described in the QS handout with the theoretical choice for B for the size of the factor base, and to apply the program to factorise the numbers n below. Printouts of your program must be included in your submissions for a grade to be assigned, but grades are dependent solely on your numerical answers.

For each number n listed below do the following.

1. List a set of exponents e_1, e_2, \dots, e_{K+2} and a set of x_j such that

$$(x_1^2 - n)^{e_1} (x_2^2 - n)^{e_2} \dots (x_{K+2}^2 - n)^{e_{K+2}}$$

is a perfect square, y^2 , and

2. such that when $x = x_1^{e_1} x_2^{e_2} \dots x_{K+2}^{e_{K+2}}$ and y is as above $\gcd(x \pm y, n)$ gives a non-trivial factorisation of n , 3. and list the values of x, y and $\gcd(x \pm y, n)$.

$$\begin{aligned} n &= 3215031751, \\ n &= 9912409831, \\ n &= 576460765725196321, \\ n &= 1152921515344265237, \\ n &= 31868712526338419047. \end{aligned}$$

It should be possible to copy these numbers from this .pdf. They can also be copied from my web site.

<https://personal.science.psu.edu/rcv4/467f25/467f25.html>

For several of these numbers it may be necessary to increase the number of B -factorable numbers from $K + 2$ to maybe $K + 8$. For the last number, if you are using Pari/gp you will need to be careful about memory, the allotment of which can be increased by `allocatemem`, and it may be necessary to choose something somewhat smaller than B^2 for the initial choice of the number of x to try.