MATH 467 FACTORIZATION AND PRIMALITY TESTING, FALL 2025, PRACTICE EXAM 2.

Mid-term Exam 2 will on Monday 3rd November. 9:05-9:55, 133 Erickson.

1. Show that 2 is a primitive root modulo 11 and draw up a table of discrete logarithms to this base modulo 11. Hence, or otherwise, find all solutions to the following congruences.
   (i) $x^6 \equiv 7 \pmod{11}$,
   (ii) $x^{48} \equiv 9 \pmod{11}$,
   (iii) $x^7 \equiv 8 \pmod{11}$.

2. Let g be a primitive root modulo p. Prove that no k exists satisfying $g^{k+2} \equiv g^{k+1} + 1 \equiv g^k + 2 \pmod{p}$.

3. Find all primes p such that $x^2 \equiv 13 \pmod{p}$ has a solution.

4. Evaluate the following Legendre symbols, showing your working (i) $\left(\frac{-1}{103}\right)_L$, (ii) $\left(\frac{2}{103}\right)_L$, (iii) $\left(\frac{7}{103}\right)_L$.