Math 467 Factorization & Primality Testing, Solutions 4

Find a non–trivial factor of (1) 19109, (2) 2048129, (3) 9912409831.

19109		2048129					
d	r	d	r				
2	1	2	1	$t = 1$	$x = 2863$	$t = 47$	$x = 19623$
3	2	3	2	$t = 1$	$x = 2864$	$t = 49$	$x = 20036$
5	4	5	4	$t = 1$	$x = 2865$	$t = 60$	$x = 22171$
7	6	7	6	$t = 2$	$x = 4048$	$t = 61$	$x = 22355$
11	2	11	6	$t = 2$	$x = 4049$	$t = 68$	$x = 23603$
13	12	13	5	$t = 3$	$x = 4958$	$t = 69$	$x = 23776$
17	1	17	3	$t = 3$	$x = 4959$	$t = 71$	$x = 24118$
19	14	19	5	$t = 4$	$x = 5725$	$t = 75$	$x = 24788$
23	19	23	2	$t = 5$	$x = 6401$	$t = 78$	$x = 25279$
$t = 1$	$x = 277$	29	4	$t = 6$	$x = 7012$	$t = 80$	$x = 25601$
$t = 2$	$x = 391$	31	21	$t = 7$	$x = 7573$	$t = 82$	$x = 25919$
$(x + y, n) = 197$		37	31	$t = 8$	$x = 8096$	$t = 85$	$x = 26389$
$(x - y, n) = 97$		41	15	$t = 9$	$x = 8587$	$t = 90$	$x = 27154$
		43	39	$t = 10$	$x = 9052$	$t = 92$	$x = 27454$
		47	10	$t = 14$	$x = 10710$	$t = 95$	$x = 27898$
		53	50	$t = 15$	$x = 11086$	$t = 97$	$x = 28190$
		59	3	$t = 17$	$x = 11802$	$t = 98$	$x = 28335$
		61	54	$t = 18$	$x = 12144$	$t = 103$	$x = 29049$
		67	6	$t = 20$	$x = 12801$	$t = 109$	$x = 29883$
		71	63	$t = 21$	$x = 13117$	$t = 111$	$x = 30156$
		73	41	$t = 23$	$x = 13727$	$t = 121$	$x = 31485$
		79	54	$t = 26$	$x = 14595$	$t = 123$	$x = 31744$
		83	21	$t = 27$	$x = 14873$	$t = 126$	$x = 32129$
		89	61	$t = 28$	$x = 15146$	$(x + y, n) = 16127$	
		97	71	$t = 29$	$x = 15414$	$(x - y, n) = 127$	
		101	51	$t = 34$	$x = 16690$		
		103	77	$t = 36$	$x = 17174$		
		107	42	$t = 39$	$x = 17875$		
		109	19	$t = 45$	$x = 19201$		
		113	4	$t = 46$	$x = 19413$		

(3). $t = 4$, $x = 398245$, $\gcd(x + y, n) = 49871$, $\gcd(x - y, n) = 198761$.

Of course in example (2) trial division would have found the factor 127 more quickly, but you only know that after the event!

In example (3) the trial division goes out to 2143, but then the factorization is found with the 27-th t, x pair, so is quite fast.