MATH 467 FACTORIZATION AND PRIMALITY, FALL TERM 2024, PROBLEMS 3

GREATEST COMMON DIVISOR

Return by Monday 16th September

1. Write a program to find x and y such that $mx + ny = \gcd(m, n)$ where
(i) $m = 8148657527$, $n = 8148653735$,
(ii) $m = 8418785375$, $n = 7849911069$,
(iii) $m = 4029583209458450398503$, $n = 3449459408504500003009$,
(iv) $m = 304250263527210$, $n = 230958203482321$.
A copy of your program should be submitted with your solutions to gain credit.

2. Show that if $\gcd(a, b) = 1$, then $\gcd(a - b, a + b) = 1$ or 2. Exactly when is the value 2?

3. The Fibonacci sequence (1202) is defined iteratively by $F_1 = F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$ $(n = 2, 3, \dots)$. Show that if $m, n \in \mathbb{N}$ satisfy $m|F_n$ and $m|F_{n+1}$, then $m = 1$.

4. The squarefree numbers are the natural numbers which have no repeated prime factors, e.g 6, 105. Note that 1 is the only natural number which is both squarefree and a perfect square. Prove that every $n \in \mathbb{N}$ with $n > 1$ can be written uniquely as the product of a perfect square and a squarefree number.

5. Let $a \in \mathbb{N}$ and $b \in \mathbb{Z}$. Prove that the equations $\gcd(x, y) = a$ and $xy = b$ can be solved simultaneously in integers x and y if and only if $a^2|b$.