MATH 467 FACTORIZATION AND PRIMALITY TESTING, FALL 2024, PRACTICE EXAM 2.

Mid-term Exam 2 will on Wednesday 30th October. 9:05-9:55, 012 Walker.

1. Suppose that p, q and r are distinct primes. Prove that

$$p^{(q-1)(r-1)} + q^{(r-1)(p-1)} + r^{(p-1)(q-1)} \equiv 2 \pmod{pqr}.$$

2. Solve the simultaneous congruences

$$x \equiv 4 \pmod{19}$$
$$x \equiv 5 \pmod{31}$$

3. Show that 2 is a primitive root modulo 11 and draw up a table of discrete logarithms to this base modulo 11. Hence, or otherwise, find all solutions to the following congruences.
   (i) $x^6 \equiv 7 \pmod{11}$,
   (ii) $x^{48} \equiv 9 \pmod{11}$,
   (iii) $x^7 \equiv 8 \pmod{11}$.

4. Evaluate the following Legendre symbols, showing your working (i) $\left(\frac{-1}{103}\right)_L$, (ii) $\left(\frac{2}{103}\right)_L$, (iii) $\left(\frac{7}{103}\right)_L$.