

**MATH 467 FACTORIZATION AND PRIMALITY
TESTING, FALL 2023, PROBLEMS 5**

Return by Monday 2nd October

Congruences

1. Eggs in basket problem (India 7c.). Find the smallest number of eggs such that when eggs are removed 2, 3, 4, 5 or 6 at a time 1 remains, but when eggs are removed 7 at a time none remain.
2. Let $f(x)$ denote a polynomial of degree at least 1 with integer coefficients and positive leading coefficient.
 - (i) Show that if $f(x_0) = m > 0$, then $f(x) \equiv 0 \pmod{m}$ whenever $x \equiv x_0 \pmod{m}$.
 - (ii) Show that there are infinitely many $x \in \mathbb{N}$ such that $f(x)$ is not prime.
3. Find all solutions (if there are any) to each of the following congruences
 - (i) $x^2 \equiv -1 \pmod{7}$, (ii) $x^2 \equiv -1 \pmod{13}$, (iii) $x^5 + 4x \equiv 0 \pmod{5}$.
4. First find a primitive root modulo 19 and then find all primitive roots modulo 19.
5. The Carmichael function $\lambda(m)$ is the smallest positive number such that

$$\text{ord}_a(m) \mid \lambda(m)$$

whenever $(a, m) = 1$. Prove that $\lambda(m) \mid \phi(m)$.