MATH 467 FACTORIZATION AND PRIMALITY TESTING, FALL TERM 2023, PROBLEMS 4

Return by Monday 25th September

Congruences

1. Solve where possible.
(i) $91x \equiv 84 \pmod{143}$
(ii) $91x \equiv 84 \pmod{147}$

2. Prove that $7n^3 - 1$ can never be a perfect square.

3. Suppose that $m_1, m_2 \in \mathbb{N}$, $(m_1, m_2) = 1$, $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$ if and only if $a \equiv b \pmod{m_1 m_2}$.

4. Write a program to compute $2^n \pmod{n}$ and apply it to 12341137 and 12341141 to determine which one is certainly composite. Include a copy of your program.

5. Solve the simultaneous congruences

$$
\begin{aligned}
x &\equiv 3 \pmod{6} \\
x &\equiv 5 \pmod{35} \\
x &\equiv 7 \pmod{143} \\
x &\equiv 11 \pmod{323}
\end{aligned}
$$