

**MATH 467 FACTORIZATION AND  
PRIMALITY, FALL TERM 2023, PROBLEMS 2**

GREATEST COMMON DIVISOR

*Return by Wednesday 6th September*

1. Find integers  $x$  and  $y$  such that  $1547x + 2197y = (1547, 2197)$ .
2. Write a program to find  $x$  and  $y$  such that  $mx + ny = \gcd(m, n)$  where
  - (i)  $m = 8148657527$ ,  $n = 8148653735$ ,
  - (ii)  $m = 8418785375$ ,  $n = 7849911069$ .A copy of your program should be submitted with your solutions to gain credit.
3. Let  $a, b, c \in \mathbb{Z}$  with  $a$  and  $b$  not both zero. Prove each of the following.
  - (i) If  $(a, b) = 1$  and  $a|bc$ , then  $a|c$ .
  - (ii)  $\left(\frac{a}{(a,b)}, \frac{b}{(a,b)}\right) = 1$ .
  - (iii)  $(a, b) = (a + cb, b)$ .
4. Show that if  $(a, b) = 1$ , then  $(a - b, a + b) = 1$  or  $2$ . Exactly when is the value  $2$ ?
5. Show that if  $ad - bc = \pm 1$ , then  $(a + b, c + d) = 1$ .