

**Example 6.4.** *Sums of two squares have a remarkable multiplicative property. Consider the following table.*

2	$1^2 + 1^2$	26	$1^2 + 5^2$	68	$2^2 + 8^2$	100	$6^2 + 8^2$
4	$0^2 + 2^2$	29	$2^2 + 5^2$	72	$6^2 + 6^2$	104	$2^2 + 10^2$
5	$1^2 + 2^2$	34	$3^2 + 5^2$	74	$5^2 + 7^2$	106	$5^2 + 9^2$
8	$2^2 + 2^2$	40	$2^2 + 6^2$	80	$4^2 + 8^2$	116	$4^2 + 10^2$
9	$0^2 + 3^2$	45	$3^2 + 6^2$	81	$0^2 + 9^2$	117	$6^2 + 9^2$
10	$1^2 + 3^2$	50	$5^2 + 5^2$	82	$1^2 + 9^2$	122	$1^2 + 11^2$
13	$2^2 + 3^2$	52	$4^2 + 6^2$	85	$2^2 + 9^2$	125	$5^2 + 10^2$
20	$2^2 + 4^2$	58	$3^2 + 7^2$	90	$3^2 + 9^2$	128	$8^2 + 8^2$
25	$0^2 + 5^2$	65	$1^2 + 8^2$	98	$7^2 + 7^2$	130	$3^2 + 11^2$

This looks as though, if a number  $n$  has a factorisation  $ab$  with both  $a$  and  $b$  being sums of two squares, then  $n$  is also the sum of two squares.

For example  $130 = 2 \times 5 \times 13$  and both 2, 5 and 13 are sums of two squares.