The basic results on sums of squares depend on the theory of quadratic residues, so this chapter is a natural continuation of the previous one.

1	0^2+1^2	$0^2+0^2+0^2+1^2$	13	2^2+3^2	$0^2+0^2+3^2+3^2$
2	1^2+1^2	$0^2+0^2+1^2+1^2$	17	1^2+4^2	$0^2+0^2+1^2+4^2$
3		$0^2+1^2+1^2+1^2$	19		$1^2+1^2+1^2+4^2$
4	0^2+2^2	$0^2+0^2+0^2+2^2$	23		$1^2+2^2+3^2+3^2$
5	1^2+2^2	$0^2+0^2+1^2+2^2$	29	2^2+5^2	$0^2+0^2+2^2+5^2$
6		$0^2+1^2+1^2+2^2$	31		$1^2+1^2+2^2+5^2$
7		$1^2+1^2+1^2+2^2$	37	1^2+6^2	$1^2+1^2+1^2+2^2$
8	2^2+2^2	$0^2+0^2+2^2+2^2$	41	4^2+5^2	$0^2+0^2+4^2+5^2$
9	0^2+3^2	$0^2+1^2+2^2+2^2$	43		$1^2+1^2+4^2+5^2$
10	1^2+3^2	$0^2+0^2+1^2+3^2$	47		$1^2+1^2+3^2+6^2$
11		$0^2+1^2+1^2+3^2$	53	2^2+7^2	$0^2+0^2+2^2+7^2$
12		$1^2+1^2+1^2+3^2$	59		$0^2+1^2+3^2+7^2$

So it looks like every number is the sum of four squares and it seems that the primes $p \equiv 1 \pmod 4$ always have a representation, but those $\equiv 3 \pmod 4$ never have one. But what about general n? Fermat found a rule which tells us precisely which numbers are the sum of two squares. Eventually Lagrange proved the four square theorem.