# MATH 465 NUMBER THEORY, SPRING 2025, PROBLEMS 7

*Return by Monday 3rd March*

1. Prove that if $p$ is an odd prime, then

$$\sum_{x=1}^{p}\sum_{y=1}^{p}\left(\frac{xy+1}{p}\right)_L = p.$$

2. Suppose that $p \nmid a$. Show that the number of solutions to $ax^2+bx+c \equiv 0 \pmod{p}$ is $1+\left(\frac{b^2-4ac}{p}\right)_L$.

3. Let $g$ be a primitive root modulo $p$. (i) Prove that the quadratic residues are precisely the residue classes $g^{2k}$ with $0 \le k < \frac{1}{2}(p-1)$.

(ii) Show that the sum of the quadratic residues modulo $p$ is the 0 residue.

4. Recall that for every reduced residue class $r$ modulo $p$ there is a unique reduced residue class $s_r$ modulo $p$ such that $1 \equiv rs_r \pmod{p}$, and that for every reduced residue class $s$ modulo $p$ there is a unique $r$ such that $s_r \equiv s \pmod{p}$. Hence prove that if $p$ is an odd prime, then

$$\sum_{r=1}^{p-1}\left(\frac{r(r+1)}{p}\right)_L = \sum_{r=1}^{p-1}\left(\frac{1+s_r}{p}\right)_L = \sum_{s=1}^{p-1}\left(\frac{1+s}{p}\right)_L = -1.$$