**MATH 465 NUMBER THEORY,**
**SPRING TERM 2025, PROBLEMS 4**

*Return by Monday 10th February*

1. Prove that when a natural number is written in the usual decimal notation, (i) it is divisible by 3 if and only if the sum if its digits is divisible by 3 and (ii) it is divisible by 9 if and only if the sum if its digits is divisible by 9.

2. Solve $11x \equiv 21 \pmod{91}$.

3. (i) Prove that $x^3$ lies in one of the residue classes $\overline{0}$, $\overline{1}$, $\overline{8}$ modulo 9.
   (ii) Prove that if $n$ is in one of the residue classes $\overline{4}$ or $\overline{5}$ modulo 9, then

$$x^3 + y^3 + z^3 = n$$

has no solution in integers $x, y, z$.

4. Prove that if $m$, $n \in \mathbb{N}$ and $(m, n) = 1$, then $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$.