## MATH 465 NUMBER THEORY, SPRING TERM 2025, PRACTICE EXAM 3

**Note: Mid-term Exam 3 will be 1:25 on Wednesday 9th April in room 216 Thomas.**

1. Evaluate the following Legendre symbols, showing your working (i) $\left(\frac{-1}{103}\right)_L$, (ii) $\left(\frac{2}{103}\right)_L$, (iii) $\left(\frac{7}{103}\right)_L$.

2. Given that 4999 is prime, determine the number of solutions of the congruence $x^2 \equiv 2021 \pmod{4999}$.

3. Suppose that $p \equiv 1 \pmod 6$. (i) Prove that the congruence $z^2 \equiv -3 \pmod p$ is soluble in $z$.
(ii) Prove that there is an $m$ with $m = 1$, 2 or 3 such that $x^2 + 3y^2 = mp$ is soluble in integers $x$ and $y$.
(iii) Deduce that there are integers $x$ and $y$ such that $x^2 + 3y^2 = p$.

4. Prove that for every positive integer $n$

$$\sum_{m|n} \mu(m)d(m) = (-1)^{\omega(n)}$$

where $\omega(n)$ is the number of different prime factors of $n$.