MATH 465 NUMBER THEORY, SPRING TERM 2025, PRACTICE EXAM 2

Note: Mid-term Exam 2 will be 1:25 on Wednesday 5th March in room 216 Thomas.

1. Suppose that p, q and r are distinct primes. Prove that
$$p^{(q-1)(r-1)} + q^{(r-1)(p-1)} + r^{(p-1)(q-1)} \equiv 2 \pmod{pqr}.$$

2. Solve the simultaneous congruences
$$x \equiv 4 \pmod{19}$$
$$x \equiv 5 \pmod{31}$$

3. Show that 2 is a primitive root modulo 11 and draw up a table of indices to this base modulo 11. Hence, or otherwise, find all solutions to the following congruences.
(i) $x^6 \equiv 7 \pmod{11}$,
(ii) $x^{48} \equiv 9 \pmod{11}$,
(iii) $x^7 \equiv 8 \pmod{11}$.

4. Given that 4327 is prime, determine the number of solutions of the congruence
$x^2 \equiv 2021 \pmod{4327}$.