MATH 465 NUMBER THEORY, SPRING TERM 2025, PRACTICE EXAM 1.

Note: Exam 1 will be 1:25–2:15, Wednesday 5th February 2025 in 216 Thomas

1. (25 marks) Suppose that $l, m, n \in \mathbb{N}$. Prove that $(lm, ln) = l(m, n)$.

2. (25 marks) (i) Show that if $(l, 6) = 1$, then $6|l \pm 1$.
   (ii) Show that if $6|l - 1$ and $6|m - 1$, then $6|lm - 1$.
   (iii) Show that if $6|lm + 1 \pmod 6$, then either $6|l + 1$ or $6|m + 1$.
   (iv) Show that if $n \in \mathbb{N}$ and $6|n + 1$, then there is a prime number p such that $p|n$ and $6|p + 1$.
   (v) Show that there are infinitely many primes of the form $6k - 1$.

3. (25 marks) Find all pairs of integers x and y such that $922x + 2163y = 7$.

4. (25 marks) (i) Prove that if $x \in \mathbb{Z}$, then $4|x^2$ or $4|x^2 - 1$.
   (ii) Prove that $5y^2 + 2 = z^2$ has no solutions with $y, z \in \mathbb{Z}$.