

**MATH 421 COMPLEX ANALYSIS, FALL  
TERM 2004, PRACTICE FINAL EXAM**

1. Sketch the following circlines, giving the centre and radius of those which are circles. (i)  $|z + i| = |z - 3i|$ , (ii)  $|z + 1| = 4|z - 1|$ .
2. Suppose  $R$  is a region and  $z \in R$ . Prove that  $R \setminus \{z\}$  is a region.
3. Show that  $f'(z)$  exists at no points of  $\mathbb{C}$  when  $f(x + iy) = \exp(-y - ix)$ .
4. Evaluate  $\int_{\mathcal{C}} (1 - z^{-2}) \exp(z + \frac{1}{z}) dz$  along any path  $\mathcal{C}$  leading from  $-1$  to  $1$  which does not pass through the origin.
5. (i) What is the largest region in which  $\frac{1}{(z^2+1)^2}$  is holomorphic? (ii) Evaluate  $\int_{\mathcal{C}} \frac{dz}{(z^2+1)^2}$  where  $\mathcal{C} = \{\frac{3}{2}i + e^{it} : 0 \leq t \leq 2\pi\}$ .
6. Find the Laurent expansion of  $\frac{e^z}{(z+1)^2}$  about  $-1$  in powers of  $z + 1$ . Write down the residue of  $\frac{e^z}{(z+1)^2}$  at  $-1$ .
7. Evaluate  $\int_0^\infty \frac{x \sin x}{x^2+4} dx$ .