

**MATH 421 COMPLEX ANALYSIS,
FALL TERM 2004, PRACTICE EXAM 1**

Note that the first exam is on Wednesday 6th October, at 9:05 in Room 109 Bouke.

1. (i) Find the absolute value of

$$\frac{(3 + 4i)(-1 + 2i)}{(-1 - i)(3 - i)}.$$

- (ii) Show that if $a \neq 0$, then $\frac{1}{a} = \frac{\bar{a}}{|a|^2}$, and find the real part of $\frac{4-3i}{-1+i}$.

2. Find the image under the Möbius transformation $z \mapsto w : w = \frac{z-1}{z+1}$, of (a) the circle $|z + 2| = 1$, (b) the line $\Re z = \Im z$.

3. Sketch the set of points z determined by the given condition.

(a) $|z - 1 - i| \neq |z + 1 + i|$,

(b) $|z - i - 2| > 3$.

Which, if any, of these sets are regions?

4. (a) Prove that $\{z : 0 < \Re z < 1\}$ is an open set in \mathbb{C} . (b) Prove that if S and T are closed sets in \mathbb{C} , then so is $S \cup T$.