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Review of Calculus The derivative Formulæ Inverse Functions Tangent Implicit differentiation Extremal values

Introduction to Analysis: Review of Calculus

Robert C. Vaughan

November 28, 2023

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Review of Calculus

- The derivative Formulæ Inverse Functions Tangent Implicit differentiation Extremal values
- Curve Sketching

• This course has relatively little to do directly with calculus.

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- The derivative Formulæ Inverse Functior Tangent Implicit
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- Analysis was largely developed in the nineteenth century by the need to understand what was meant by a limit and place the fundamental theorems on limits on a sound axiomatic basic, as had been done by Euclid and the Pythagorean school about 2000 years earlier for geometry.

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• We shall see that the idea of a limit is intimately connected with what we mean by a number.

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The derivative

Formulæ Inverse Function Tangent Implicit differentiation

• The immediate connection with calculus is that the derivative and integral are usually both defined as a result of a limiting process.

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The derivative

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- The immediate connection with calculus is that the derivative and integral are usually both defined as a result of a limiting process.
- Thus given a real valued function *f* it is usual to define the derivative *f'* by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

or if we think in terms of a curve y = f(x) in the x, y plane, then

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

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where $\delta y = y(x + \delta x) - y(x)$.

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The derivative

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• These are, of course, just symbolic representations of the same thing.

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• Example. $f(x) = x^4$.

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• Example. $f(x) = x^4$.

 $f(x+h)-f(x) = (x+h)^4 - x^4 = 4x^3h + 6x^2h^2 + 4xh^3 + h^4,$

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• Example. $f(x) = x^4$.

 $f(x+h)-f(x) = (x+h)^4 - x^4 = 4x^3h + 6x^2h^2 + 4xh^3 + h^4,$

$$\frac{f(x+h) - f(x)}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3$$

Then each of the terms 6x²h, 4xh², h³ will tend to 0 as h tends to 0 and we will conclude that in this case

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=4x^3.$$

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Formulæ Inverse Functions Tangent Implicit differentiation Extremal values • In calculus we learn, or at least see, many formulæ for derivates. Maybe you can help me out by recalling some of them.

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- In calculus we learn, or at least see, many formulæ for derivates. Maybe you can help me out by recalling some of them.
- Let $f(x) = x^n$ (*n* a non-negative whole number). What is f'(x)?

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- Inverse Function
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• What about $f(x) = x^{-1}$?

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- Let $f(x) = \sin x$. What is f'(x)?

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- Let $f(x) = \arctan x$. What is f'(x)?

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Formulæ Inverse Functions Tangent Implicit differentiation Extremal values Curve Sketching • In practice the functions we come across are much more complicated than this, so it is normal to develop some general formulæ.

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- In practice the functions we come across are much more complicated than this, so it is normal to develop some general formulæ.
- The simplest is the sum of two general functions, suppose f, g h are connected by f(x) = g(x) + h(x). Then

$$f'(x) = g'(x) + h'(x).$$

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One way of thinking of this is that differentiation is a "linear operator".

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- How about a product, f(x) = g(x)h(x)? Is there a formula for
- How about f(x) = g(h(x))?
- f(x) = 1/h(x)?

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• Example

$$y = (x^2 + 2)^{10}$$
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Formulæ Inverse Functions Tangent Implicit differentiation Extremal values • Now we come to tricky one. Let

$$f(x)=\frac{g(x)}{h(x)}.$$

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what is f'(x)?

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Now define

$$f(x) = \begin{cases} \sin(1/x) & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$
$$g(x) = \begin{cases} x \sin(1/x) & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$
$$h(x) = \begin{cases} x^2 \sin(1/x) & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

A tricky example

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 If x ≠ 0, then each of these is differentiable, by use of the chain product rules. For example

$$h'(x) = 2x\sin(1/x) - \cos(1/x) \quad (x \neq 0).$$

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A tricky example

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$$h'(x) = 2x\sin(1/x) - \cos(1/x) \quad (x \neq 0).$$

- What happens in each case when x = 0?
- How could we be sure that our guesses are correct?

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 $y = x^n$, $y' = nx^{n-1}$, $y = \sin(x), \quad y' = \cos(x),$ $v = \cos(x), \quad v' = -\sin(x),$ $y = \tan(x), \quad y' = \sec^2(x),$ $v = e^x$, $v' = e^x$. $y = \ln(x), \quad y' = 1/x,$ $y = \arctan(x), \quad y' = \frac{1}{1+x^2},$ $y = \arcsin(x), \quad y' = (1 - x^2)^{-1/2}.$ y = u + v, v' = u' + v'. v = uv, v' = u'v + uv'. $y = \frac{u}{v}, \quad y' = \frac{u'v - uv'}{v^2}.$

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• Suppose that the function

y = f(x)

is such that each y corresponds to a unique x.

Inverse Functions

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• Suppose that the function

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• Then for each y we can define a function

g(y) = x

which has the property that g(f(x)) = x. Such a function is called an inverse function.

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• Now we can apply the chain rule and obtain

$$g'(f(x))f'(x) = 1.$$

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$$g'(f(x))f'(x)=1.$$

• In other words $g'(y) = \frac{1}{f'(x)}$.

• Suppose that the function

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g(y) = x

which has the property that g(f(x)) = x. Such a function is called an inverse function.

Now we can apply the chain rule and obtain

$$g'(f(x))f'(x)=1.$$

- In other words $g'(y) = \frac{1}{f'(x)}$.
- An important example is $y = f(x) = e^x$ and $g(y) = \ln y$. Then $f'(x) = e^x = y$, so that $g(y) = \frac{1}{f'(x)} = \frac{1}{f'(x)}$.

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• 1.

 $y = x^5 - x^4 + x^2$ $v' = 5x^4 - 4x^3 + 2x.$

Some examples

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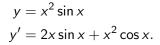
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Some examples

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• 1.

• 2.

 $y = x^5 - x^4 + x^2$ $v' = 5x^4 - 4x^3 + 2x$.

 $y = x^2 \sin x$ $y' = 2x \sin x + x^2 \cos x.$

• 2.

$$y = \frac{1-x}{1+x}$$

$$y' = \frac{(1+x)(-1) - (1-x) \cdot 1}{(1+x)^2} = \frac{-2}{(1+x)^2}.$$

Equation of the tangent

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• It is sometimes useful in applications to know the tangent to a point $(x_0, f(x_0))$ on a curve y = f(x).

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- It is sometimes useful in applications to know the tangent to a point (x₀, f(x₀)) on a curve y = f(x).
- The derivative at the point is $f'(x_0)$ and is the slope of the curve at that point, so will also be the slope of the tangent.

Equation of the tangent

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Equation of the tangent

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- The derivative at the point is $f'(x_0)$ and is the slope of the curve at that point, so will also be the slope of the tangent.
- The equation of a line through the point (x₀, y₀) with slope *m* is given by

$$y-y_0=m(x-x_0).$$

substituting from above gives

$$y - f(x_0) = f'(x_0)(x - x_0)$$

which we can rearrange to give

$$y = f'(x_0)x + f(x_0) - x_0f'(x_0)$$

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• Something which is sometimes useful but often isn't covered in calculus courses is implicit differentiation.

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- Something which is sometimes useful but often isn't covered in calculus courses is implicit differentiation.
- Suppose that there is no simple formula connection x and y = f(x). For example $x^3 + y^3 2x + 3y = 0$.

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- Something which is sometimes useful but often isn't covered in calculus courses is implicit differentiation.
- Suppose that there is no simple formula connection x and y = f(x). For example $x^3 + y^3 2x + 3y = 0$.
- Obviously we can differentiate both sides and obtain

$$3x^2 + 3y^2 \frac{dy}{dx} - 2 + 3\frac{dy}{dx} = 0.$$

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• Then we can solve for the derivative to obtain

$$\frac{dy}{dx} = \frac{2-3x^2}{3y^2+3}$$

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$$\frac{dy}{dx} = \frac{2-3x^2}{3y^2+3}.$$

• The point (2, -1) lies on the curve. Thus

$$\frac{dy}{dx}_{(x,y)=(2,-1)} = -\frac{5}{3}$$

and the tangent at (2, -1) is given by $y = -\frac{5}{3}x + \frac{7}{3}$.

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• If a curve does not have slope 0 at a particular point, then it cannot be a local maximum or minimum.

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- Thus to find candidate extremal points we can first check for points for which the derivative is 0.

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- Extremal values

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- However if f'(x) = 0 when $x = x_0$ AND f'(x) > 0 just to the left of x_0 and f'(x) < 0 just to the right of x_0 , then f must have a local maximum at x_0 .

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• If these inequalities are reversed, then we have a local minimum.

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• Example.
$$y = x^3 - x$$
. Then $\frac{dy}{dx} = 3x^2 - 1$ and this is 0 when $x = \frac{\pm 1}{\sqrt{3}}$.

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• If $x < -1/\sqrt{3}$, then $x^2 > 1/3$, $\frac{dy}{dx} > 0$, and if $-1/\sqrt{3} < x < 0$, then $x^2 < 1/3$ and so $\frac{dy}{dx} < 0$.

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- If $x < -1/\sqrt{3}$, then $x^2 > 1/3$, $\frac{dy}{dx} > 0$, and if $-1/\sqrt{3} < x < 0$, then $x^2 < 1/3$ and so $\frac{dy}{dx} < 0$.
- Thus $x^3 x$ has a local maximum at $-1/\sqrt{3}$.

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- These inequalities are reversed in the neighbourhood of $1/\sqrt{3}$, so $x^3 x$ has a local minimum at $1/\sqrt{3}$.

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- These inequalities are reversed in the neighbourhood of $1/\sqrt{3}$, so $x^3 x$ has a local minimum at $1/\sqrt{3}$.
- Alternatively one can check $\frac{d^2y}{dx^2} = 6x$. At $x = \frac{\pm 1}{\sqrt{3}}$ this is $\pm 2\sqrt{3}$.

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- Note that sometimes computing the second derivative can be a right pain and the above method is usually easier.

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Curve Sketching

• Armed with our understanding of derivatives it is often useful in applications to sketch a relevant curve.

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$$y = x^3(x-1)^2$$
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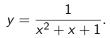
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- Armed with our understanding of derivatives it is often useful in applications to sketch a relevant curve.
- Example. $y = x^3(x-1)^2$.
- Second Example.

$$y = \frac{1}{x^2 + x + 1}.$$

• Third example.

$$y=\frac{x^2-1}{x(x+2)}.$$

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