

**MATH 401 INTRODUCTION TO ANALYSIS-I,
SPRING TERM 2024, PROBLEMS 14**

Return by Monday 22nd April

1. Suppose that $f : \mathbb{R} \mapsto \mathbb{R} : f(x) = |x|$. Show that f is uniformly continuous on \mathbb{R} .
2. Let $f : \mathbb{R}^+ \mapsto \mathbb{R}^+ : x \mapsto x^{1/2}$. Prove that f is differentiable at every $x \in \mathbb{R}^+$.
3. Prove that $\sin x$ is differentiable at every $x \in \mathbb{R}$ and that $\sin'(x) = \cos x$.
4. Let $f : \mathbb{R} \mapsto \mathbb{R}$ be defined by $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(x) = 0$ when $x = 0$. Prove that f is differentiable at 0.
5. Prove that $\log'(x) = 1/x$.