

**MATH 401 INTRODUCTION TO ANALYSIS-I,  
SPRING TERM 2024, PROBLEMS 13**

*Return by Monday 15th April*

1. Let  $\exp(x) : x \mapsto \mathbb{R}$  be the function defined in (6.9).
  - (i) Prove that  $\exp(x)$  is continuous at 0.
  - (ii) Prove that  $\exp(x)$  is continuous at every  $\xi \in \mathbb{R}$ .
  - (iii) Prove that  $\log(x)$  is continuous at every  $\eta \in \mathbb{R}^+$ .
2. Prove that  $f : \mathbb{R} \mapsto \mathbb{R} : f(x) = |x|$  is continuous at all  $\xi \in \mathbb{R}$ .
3. Define the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x^2$  when  $x \in \mathbb{Q}$  and  $f(x) = x$  when  $x \in \mathbb{R} \setminus \mathbb{Q}$ . Prove that  $f$  is continuous at exactly two points.
4. Prove that the equation  $x^{3456} + x^{1234} - 1 = 0$  has a solution in  $[0, 1]$ .
5. Prove that the curve  $y = x^2$  intersects the cubic curve  $y = x^4 - 2x^2 - x + 1$  in at least four distinct points.