

**MATH 401 INTRODUCTION TO ANALYSIS-I,
SPRING TERM 2024, PROBLEMS 8**

Return by Monday 11th March

1. Prove, using the definition of a limit, that

$$\lim_{n \rightarrow \infty} \frac{n}{n^2 + 1} = 0.$$

2. Let c be a fixed positive number and

$$a_n = \frac{1}{1 + nc} \text{ where } c > 0.$$

Use the definition of a limit to prove that $\langle a_n \rangle$ converges.

3. Prove that the sequence $\langle n^{1/3} \rangle$ diverges.
4. Suppose that the sequence $\langle a_n \rangle$ converges to l . Let

$$s_n = \frac{a_{n+1} + a_{n+2} + \cdots + a_{2n}}{n}.$$

Using only the definition of limit, prove that $\langle s_n \rangle$ converges to l .

5. Prove that

$$\lim_{n \rightarrow \infty} \frac{3n^5 - 4n^3 + 2n + 7}{4n^5 + 5n^4 + 6n^3 + n^2 + 1} = \frac{3}{4}.$$