# MATH 401 INTRODUCTION TO ANALYSIS, SPRING TERM 2024, PROBLEMS 7

*Return by Monday 26th February*

1. Let $\mathcal{U} = \left\{ \frac{2n+1}{n+1} : n \in \mathbb{N} \right\}$.
   (i) Prove that $\mathcal{U}$ is non-empty and bounded above by 2.
   (ii) Prove that if $a$ is a real number with $a < 2$, then there is an $n \in \mathbb{N}$ such that $a < \frac{2n+1}{n+1}$.
   (iii) Prove that $\sup \mathcal{U} = 2$.

2. Prove that for all $n \geq 7$ we have $3^n \leq n!$.

3. Let $a_n = \frac{n+1}{n}$. Prove that $\lim_{n \to \infty} a_n = 1$.