

**MATH 401 INTRODUCTION TO ANALYSIS,
SPRING TERM 2024, PROBLEMS 6**

Return by Monday 19th February

1. Suppose that \mathcal{A} is bounded above, $\mathcal{B} \subset \mathcal{A}$ and \mathcal{B} is non-empty. Prove that $\sup \mathcal{B}$ exists and $\sup \mathcal{B} \leq \sup \mathcal{A}$.
2. Let $\mathcal{A} = \{x : x \in \mathbb{Q}, x^3 < 5\}$. Prove that $\sup \mathcal{A}$ exists. Guess the value of $\sup \mathcal{A}$.
3. Suppose that $x \geq -1$.
 - (i) Prove that $(1+x)^2 \geq 1+2x$.
 - (ii) Prove that $(1+x)^3 \geq 1+3x$.
 - (iii) Prove that $(1+x)^4 \geq 1+4x$.Guess a general inequality for $(1+x)^n$ when $x \geq -1$ and $n \in \mathbb{N}$.