MATH 401 INTRODUCTION TO ANALYSIS,
SPRING TERM 2024, PROBLEMS 5

Return by Monday 12th February

1. Decide in each of the following cases whether or not the given set is bounded above. For those which are bounded above give three different upper bounds including the smallest one.

(i) $[3, \infty)$, (ii) $(-7, 3)$, (iii) $(-\infty, 0)$, (iv) $[-7, 3]$, (v) $\{-4, -2, 1, 5, 6, 9\}$, .

2. Give an example of a non-empty set which has least upper bound 1 but contains no element x satisfying $x < 1$.

3. Let $\mathcal{A} = \{x : x^3 + 3x^2 + 2x < 0\}$. Prove that this set is non-empty and bounded above. What is the least upper bound? Is it bounded below?

4. A set of real numbers $\mathcal{A}$ has a maximum when there is an element $m \in \mathcal{A}$ such that for every $x \in \mathcal{A}$ we have $x \leq m$.

Let a be any element of the open interval $(0, 1)$.
(i) Show that there is another $b \in (0, 1)$ with $b > a$.
(ii) Prove that $(0, 1)$ has no maximum.
(iii) Does $(0, 1)$ have a supremum?