MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2024, PROBLEMS 4

INEQUALITIES, MODULUS

Return by Monday 5th February

1. Find all real values of x such that

$$\frac{x+1}{x^2+3} < \frac{2}{x}.$$

2. (i) Prove that, for any real number a, $|a|^2 = a^2$.

(ii) Let a and b be real numbers. Show that |a + b| = |a| + |b| if and only if $ab \ge 0$.

3. Suppose that a, b, x, y are real numbers satisfying a < x < b and a < y < b. Show that |x - y| < b - a.

4. Sketch the graph of the equation y = |x| - |x - 1|.

5. Find all x such that |x + 1| + |x - 2| = 7.