MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2024, PROBLEMS 3

INEQUALITIES

Return by Monday 29th January

Summary of order axioms from class (slightly different from the Binmore):
There is a relation "$<$" which satisfies the following axioms. a, b, c denote real numbers.

O1. Exactly one of $a < b$, $a = b$, $b < a$ holds.
O2. If $a < b$ and $b < c$, then $a < c$.
O3. If $a < b$, then $a + c < b + c$ for all c.
O4. If $a < b$ and $0 < c$, then $ac < bc$.

The expression $a > b$ means $b < a$. We also use $a \leq b$ to mean "either $a < b$ or $a = b$". You may assume that $0 < 2$.

Note that the conclusions of earlier questions below are useful in answering later questions.

1. Let x be a real number with $0 < x$. Prove that $0 < x^2$.

2. Let x be a real number with $x < 0$. Prove that $0 < -x$. Deduce that $0 < x^2$. (You may suppose that $(-x)^2 = x^2$.)

3. Let x be a real number. Prove that $0 \leq x^2$.

4. Prove that if $x > 0$ and $y > 0$, then

$$(x - y)^2 < x^2 + y^2 < (x + y)^2.$$

5. Prove that for any real numbers a, b, c, d,

$$4abcd \leq a^4 + b^4 + c^4 + d^4.$$