MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2024, PROBLEMS 2

Rational Numbers and Sets

Return by Monday 22nd January

1. Prove that there is no rational number whose square is 5.
2. Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be three sets. Prove that $\mathcal{A} \cup (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \cup \mathcal{B}) \cap (\mathcal{A} \cup \mathcal{C})$.
3. Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be three sets. Prove that $\mathcal{A} \setminus (\mathcal{B} \cup \mathcal{C}) = (\mathcal{A} \setminus \mathcal{B}) \cap (\mathcal{A} \setminus \mathcal{C})$.
4. Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be three sets. Prove that

$$\big((\mathcal{B} \cap \mathcal{C}) \cup (\mathcal{C} \cap \mathcal{A})\big) \cup (\mathcal{A} \cap \mathcal{B}) = \big((\mathcal{B} \cup \mathcal{C}) \cap (\mathcal{C} \cup \mathcal{A})\big) \cap (\mathcal{A} \cup \mathcal{B}).$$