MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2024, PRACTICE EXAM 3

Note that the third exam is on Wednesday 10th April, at 1:25 in Room 011 Huck.

1. Decide the convergence of the each of the following series, in each case stating which tests you use.

$$\text{(i)} \sum_{n=1}^{\infty} \frac{3}{n^3+2} \quad \text{(ii)} \sum_{n=1}^{\infty} \frac{4}{3n+2} \quad \text{(iii)} \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}(26)^n$$

$$\text{(iv)} \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}(28)^n \quad \text{(v)} \sum_{n=1}^{\infty} (-1)^{n-1} n^{-1/4} \quad \text{(vi)} \sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)(-1)^n.$$

2. Prove, using only the definition of limit, that $\lim_{x\to 1}(5x-3) = 2$.

3. Evaluate the following limits, justifying your conclusion.

$$\text{(i)} \lim_{x\to 3} \frac{x^3+5x+7}{x^4+6x^2+8} \quad \text{(ii)} \lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}.$$

4. Define $f : (-1,1) \mapsto \mathbb{R} : f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$. Prove that if $\xi \in (-1,1)$, then

$$\lim_{x\to\xi} f(x) = f(\xi).$$