MATH 401 INTRODUCTION TO ANALYSIS-I, SPRING TERM 2024, PRACTICE EXAM 1

Note that the first exam is on Wednesday 7th February, at 1:25 in Room 011 Huck Life Sciences.

1. Differentiate with respect to x.
(i) $(2x^3 - 1)\sin x$
(ii) $\frac{7x-4}{\ln x}$

2. The function f is defined by $f(x) = \frac{4}{x-2} - \frac{1}{x+1}$. (i) Show that $f'(x) = -\frac{3x(x+4)}{(x-2)^2(x+1)^2}$ and find the maxima and minima of f. (ii) Sketch the graph of f.

3. Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be sets. Prove that $(\mathcal{A} \cap \mathcal{B}) \cup \mathcal{C} = (\mathcal{A} \cup \mathcal{C}) \cap (\mathcal{B} \cup \mathcal{C})$.

Summary of order axioms from class (slightly different from the textbook): There is a relation "$<$" which satisfies the following axioms. a, b, c denote real numbers.

O1. Exactly one of $a < b$, $a = b$, $b < a$ holds.
O2. If $a < b$ and $b < c$, then $a < c$.
O3. If $a < b$, then $a + c < b + c$ for all c.
O4. If $a < b$ and $0 < c$, then $ac < bc$.

The expression $a > b$ means $b < a$. We also use $a \leq b$ to mean "either $a < b$ or $a = b$" and $a \geq b$ to mean $b \leq a$.

4. Suppose that a, b, c are three real numbers and $0 < abc$. Prove that at least one of a, b, c is positive.