①

x_1	x_2	-3	$-x_5$	
1	-2	3	4	$= x_4$
1	-2	0	3	$= -x_2$
0	2	-3	0	$\to$ max

x_1	x_2	x_5	1	
1	-2	-4	-9	$= x_4$
+1	-1	-3	0	$= 0$
0	-2	0	-9	$\to$ min

Standard

$m \geq 0$

x_1	x_2	x_5	1	
1*	-2	-4	9	$= x_4$
1	-1	-3	0	$= m$
0	-2	0	9	$\to$ min

x_4	x_2	x_5	1	
1	2	4	9	$= x_1$
1	1	1	9	$= m$
0	-2	0	9	$\to$ min

Now we have a feasible table, but we can see that x_2 is a bad column. As such, the problem is unbounded.

$\max = \infty$

②

x_1	$-x_2$	1	$2x_4$	
1	-2	3	4	$= x_3$
1	2	0	-1	$= -x_5$
0	-2	-3	-2	$\to$ min

x_1	x_2	x_4	1	
1	2	8	3	$= x_3$
-1	2	2	0	$= x_5$
0	2	-4	-3	$\to$ min

$\Rightarrow$ Standard, feasible form. We see x_4 is a bad column. $\therefore$ Unbounded.

$\min = -\infty$

(3)

2	1	3	3	1 2	2	1 6	2 1	5	~~9~~ ~~8~~ ~~2~~
1	2	3	4	1 2	1	1	3	1 6	~~8~~ ~~2~~
2	2	1 6	3	1 2	1 1	3	5	2	~~9~~ ~~3~~ ~~2~~
1 6	1 1	3	1 1	2	1	2	3 0	2	~~8~~ ~~7~~ ~~1~~ 0
~~6~~	~~1~~	~~6~~	~~1~~	~~6~~ ~~4~~ ~~2~~	~~1~~	~~6~~	~~1~~ 0	~~6~~	

The # of Selected entries is $m+n-1=12$

Calculation of potentials is unnecessary, as all Columns have products only Shipped at there lowest cost.

∴ The Min cost of this System is:

$6\cdot 1 + 1\cdot 1 + 6\cdot 1 + 1\cdot 1 + (2+2+2)\cdot 1 + 1\cdot 1 + 6\cdot 1 + 2\cdot 1 + 1\cdot 6 = 35$

(4)

	A	B	C	D	E	F	G	H	I	
	(0) 6	(1) 1	(3)	(2)	(1) 2	(2)	(2)	(2)	(4)	~~9~~ ~~3~~ ~~2~~
	(1)	(1)	(3)	(1)	(1)	(5)	(1) 6	(3)	(1) 2	~~8~~ ~~2~~
	(2)	(2)	(5)	(3)	(1) 4	(1) 1	(3)	(1)	(2) 4	~~9~~ ~~7~~ ~~3~~ ~~2~~
	(2)	(3)	(1) 6	(1) 1	(2)	(1)	(2)	(0) 1	(2)	~~8~~ ~~7~~ ~~1~~
	~~6~~	~~1~~	~~6~~	~~1~~	~~6~~ ~~4~~ ~~2~~	1	~~6~~	~~1~~	~~6~~ ~~4~~	

The Columns have been labled **A-I**.
Notice that Columns **G** and **I** only have a Single Position of Cost 1, Shared in the same row.
These 1 cost positions should be filled first, to eliminate the 8 demand. Next, Columns **A** and **H** both have Cost 0, which would ideally be filled.

All positions/Columns have supply in their lowest price, except for in Column **I**. The remaining amounts of 4, is put in the next lowest cost box.

As such, this is a feasible and optimal solution.

min Cost $= 6\cdot 0 + 1\cdot 1 + 6\cdot 1 + 1\cdot 1 + 4\cdot 1 + 2\cdot 1 + 1\cdot 1 + 6\cdot 1 + 0\cdot 1 + 1\cdot 2 + 4\cdot 2$

$= 31$

(5)

$$\begin{array}{ccccc} -1 & & -1 & & -1 \\ \end{array}$$
$$\begin{bmatrix} 5 & 2 & 3 & 4 & 2^* \\ 2 & 3 & 5 & 1^* & 2 \\ 3 & 2 & 1^* & 3 & 1 \\ 1 & 0^* & 4 & 0 & 1 \\ 1^* & 5 & 2 & 1 & 6 \end{bmatrix}$$

we use the Hungarian Method to reduce the System.

$$\xrightarrow{1}$$

$$\begin{array}{c} -1 \\ -1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 4 & 2 & 2 & 4 & 1 \\ 1 & 3 & 4 & 1 & 1 \\ 2 & 2 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 5 & 1 & 1 & 5 \end{bmatrix}$$

Continuing with the Hungarian Method.

$$\xrightarrow{2}$$

$$\begin{bmatrix} 3 & 1 & 1 & 3 & 0^* \\ 0 & 2 & 3 & 0^* & 0 \\ 2 & 2 & 0^* & 3 & 0 \\ 0 & 0^* & 3 & 0 & 0 \\ 0^* & 5 & 1 & 1 & 5 \end{bmatrix}$$

$$\xrightarrow{3}$$

we are able to find the optimal Soln of 0's, Marking them with asterisks. Inputing this into the orignal problem, we find the Minimal price is $1+0+1+1+2=5$