

# Jairo Bochi

## *Curriculum Vitae*

### Contact Information

Department of Mathematics  
127 McAllister Building  
The Pennsylvania State University  
University Park, PA 16802, USA  
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### Research Interests

My research area is dynamical systems. I'm interested in several problems in differentiable ergodic theory, multiplicative ergodic theory, thermodynamical formalism, and ergodic optimization. I'm also keen on making connections with geometry, Lie theory, linear algebra, and control theory.

### Academic History

- 2001         *Doctor in Mathematics*, IMPA, Rio de Janeiro, Brazil.  
              Thesis title: *Zero Lyapunov exponents for conservative systems*  
              Thesis advisor: Marcelo Viana.
- 1997         *M.S. in Mathematics*, UFRGS, Porto Alegre, Brazil.  
              Advisor: Artur O. Lopes.
- 1996         *B.S. in Mathematics*, UFRGS, Porto Alegre, Brazil.

### Employment Record

- 2021–         Professor, Penn State.
- 2014–2021     Associate Professor, PUC-Chile.
- 2011–2013     Associate Professor, PUC-Rio.
- 2008–2011     Assistant Professor, PUC-Rio.
- 2005–2007     Assistant Professor, UFRGS, Porto Alegre.
- 2002–2004     Junior Researcher, IMPA, Rio de Janeiro.

### Research Grants (individual)

- 2021–2026     Startup grant, Eberly College of Science, Penn State University.
- 2018–2021     *Fondecyt Regular* 1180371, Conicyt, Chile.
- 2014–2017     *Fondecyt Regular* 1140202, Conicyt, Chile.
- 2013           *Produtividade em Pesquisa*, level 1D, CNPq, Brazil.
- 2012–2013     *Jovem Cientista da Nosso Estado*, FAPERJ, Brazil.
- 2008–2011     *Incentivo à Produtividade em Pesquisa para Novos Professores*, PUC-Rio, Brazil.
- 2007–2012     *Produtividade em Pesquisa*, level 2, CNPq, Brazil.

### Research Grants (teams) – since 2014

- 2018–         Anillo ACT172001 *New Trends in Ergodic Theory*, Conicyt, Chile; directed by Godofredo Iommi.

2014–2016 Anillo ACT1103 *Center for Dynamical Systems and Related Topics*, Conicyt, Chile; directed by Andrés Navas.

## Honors and Awards

2007–2011 Affiliate Member of the Brazilian Academy of Sciences.

## Lectures in International Congresses

- 2021 Workshop in Dynamical Systems and Related Topics, University Park, PA, USA.
- 2021 Hyperbolic and Symplectic Dynamics Workshop, Bochum, Germany. (Online talk).
- 2021 VI Congreso Latinoamericano de Matemáticos, Montevideo, Uruguay. (Online event). Plenary speaker.
- 2020 Workshop in Dynamical Systems and Related Topics, University Park, PA, USA. (Online event).
- 2020 New Trends in Lyapunov exponents, Lisbon, Portugal. (Online event).
- 2019 2020 Vision for Dynamics, Będlewo, Poland.
- 2018 International Congress of Mathematicians, invited session Dynamical Systems and Differential Equations, Rio de Janeiro, Brazil.
- 2018 Topics in Mathematical Physics. University of São Paulo (USP), São Paulo, Brazil.
- 2018 Workshop on Groups, Geometry and Dynamics, Universidad de la República, Montevideo, Uruguay.
- 2018 Thermodynamical formalism in Dynamical Systems. ICMS, Edinburgh, UK.
- 2018 British Mathematical Colloquium. University of Saint Andrews, UK.
- 2018 New Methods for Zimmer’s Conjecture. UCLA, Los Angeles, USA.
- 2017 Geometry, dynamics, and Anosov representations. USACH, Santiago, Chile.
- 2016 SUMA 2016: First joint meeting of Sociedad de Matemática de Chile and Unión Matemática Argentina. Valparaíso, Chile.
- 2015 International Conference Beyond Uniform Hyperbolicity. Olmué, Chile.
- 2015 Dynamische Systeme. Oberwolfach, Germany.
- 2015 III Palis–Balzan Symposium on Dynamical Systems. Paris, France.
- 2015 Workshop on Random Dynamical Systems and Multiplicative Ergodic Theorem. Banff, Canada.
- 2014 III Brazilian School on Dynamical Systems. Bento Gonçalves, Brazil.
- 2014 Ergodic Theory Workshop. University of North Carolina at Chapel Hill, USA.
- 2013 Ergodic Optimization and Related Fields. USP, São Paulo, Brazil.
- 2013 Mathematical Congress of the Americas. Guanajuato, Mexico.
- 2013 International Conference Beyond Uniform Hyperbolicity. Będlewo, Poland.
- 2013 Non-positive curvature, isometric actions and dynamics of cocycles. Cajón de Maipo, Chile.
- 2012 II Brazilian School on Dynamical Systems. University of São Paulo, São Carlos, Brazil.

- 2012 Montevideo Dynamical Systems Conference. Universidad de la República, Montevideo, Uruguay.
- 2011 LX Dynamical Systems Colloquium. Celebrating the 60th birthday of Rodrigo Bamón. Pucón, Chile.
- 2011 Workshop on Symplectic Dynamics. Institute for Advanced Study, Princeton, USA.
- 2011 International Conference on Dynamics Beyond Uniform Hyperbolicity. CIRM, Marseille, France.
- 2011 International Conference on Topological Methods on Dynamical Systems. UNICAMP, Campinas, Brazil.
- 2010 Workshop on Ergodic Theory. Institute Mittag-Leffler, Djursholm, Sweden.
- 2009 III CLAM Congreso Latino Americano de Matemáticos, Santiago, Chile.
- 2008 School and Workshop on Dynamics. Universidade de la República, Montevideo, Uruguay.
- 2006 International Symposium on Dynamical Systems, UFBA, Salvador, Brazil.
- 2006 Young Researchers Symposium, IMPA, Rio de Janeiro, Brazil.
- 2006 AMS Spring Meeting of the Western Section. San Francisco State University, USA.
- 2005 XIV Escuela Latinoamericana de Matemática. Solís, Uruguay.
- 2005 International Conference on Dynamical Systems. Angra dos Reis, Brazil.
- 2005 Colloquium of Dynamical Systems and Smooth Ergodic Theory. Bordeaux, France.
- 2004 International Conference on Dynamical Systems in Honor of J. Massera. Universidade de la República, Montevideo, Uruguay.
- 2004 Systèmes dynamiques multidimensionnels non-uniformément hyperboliques. CIRM, Luminy, France.
- 2003 Recent Trends in Dynamics III. Universidade do Porto, Portugal.
- 2003 International Workshop on Robustness and Partial Hyperbolicity. Búzios, Brazil.
- 2003 Research Trimester on Dynamical Systems. Scuola Normale Superiore di Pisa, Italy.
- 2001 Recent Trends in Dynamics. Universidade do Porto, Portugal.
- 2001 School and Workshop on Dynamical Systems. ICTP, Trieste, Italy.
- 2001 International Workshop on Dynamical Systems and Geometry in honor of Prof. Michel Herman. IMPA, Rio de Janeiro, Brazil.
- 2000 International Conference on Dynamical Systems. IMPA, Rio de Janeiro, Brazil.

## Research Visits

- Jan. 2019 Princeton University, by invitation of Clark Butler.
- Jun. 2018 University of Surrey (Guildford, UK), by invitation of Ian D. Morris.
- Jul. 2016 Yeshiva University (New York), by invitation of Anatole Katok.
- Jun. 2016 Universidad de la República (Montevideo), by invitation of Jana Rodríguez Hertz.
- Feb. 2016 Penn State University (State College), by invitation of Anatole Katok.
- Dec. 2015 Universidad de la República (Montevideo), by invitation of Rafael Potrie.

- Jul. 2015 UNICAMP (Campinas), by invitation of Eduardo Garibaldi.
- May. 2015 Universidad de la República (Montevideo), by invitation of Rafael Potrie.
- Dec. 2014 Universidad de la República (Montevideo), by invitation of Rafael Potrie.
- Sep. 2014 Pontifícia Universidade Católica do Rio de Janeiro, by invitation of Lorenzo J. Díaz.
- Aug. 2013 Universidad de Santiago de Chile, by invitation of Andrés Navas, and Pontificia Universidad Católica de Chile, by invitation of Mario Ponce.
- Feb. 2013 IMPAN, Polish Academy of Sciences (Warsaw), by invitation of Michał Rams.
- Mar. 2011 Universidad de Santiago de Chile, by invitation of Andrés Navas.
- Feb. 2011 Université de Bourgogne (Dijon), by invitation of Christian Bonatti.
- Jan. 2011 Université de Bordeaux, by invitation of Nicolas Gourmelon.
- Feb. 2010 Institute Mittag-Leffler (Djursholm), by invitation of Michael Benedicks.
- Jan.-Feb. 2009 Université de Bourgogne (Dijon), by invitation of Christian Bonatti.
- Jul. 2007 Rice University (Houston), by invitation of David Damanik.
- Jul. 2006 IMPA (Rio de Janeiro), by invitation of Artur Avila.
- Apr. 2006 Caltech (Pasadena), by invitation of Anton Gorodetski.
- Feb. 2006 Collège de France (Paris), by invitation of Jean-Christophe Yoccoz.
- Jun. 2004 Université de Paris 13 – Villetaneuse, by invitation of Bassam Fayad.
- May-Jun. 2003 Université de Paris 13 – Villetaneuse, by invitation of Bassam Fayad.
- Nov. 2002 KTH (Stockholm), by invitation of Michael Benedicks.
- May 2001 Université de Paris 7, by invitation of Håkan Eliasson.

## Scientific Production

### Published articles:

- Flexibility of Lyapunov exponents. With Anatole Katok and Federico Rodriguez Hertz. *Ergodic Theory and Dynamical Systems*, 42 (2022), 554–591.
- On emergence and complexity of ergodic decompositions. With Pierre Berger. *Advances in Mathematics*, 390 (2021), 107904.
- Extremal norms for fiber bunched cocycles. With Eduardo Garibaldi. *Journal de l'École polytechnique - Mathématiques*, 6 (2019), 947–1004.
- Anosov representations and dominated splittings. With Rafael Potrie and Andrés Sambarino. *Journal of the European Mathematical Society*, 21 (2019), no. 11, 3343–3414.
- Equilibrium states of generalised singular value potentials and applications to affine iterated function systems. With I.D. Morris. *Geometric and Functional Analysis*, 28 (2018), no. 4, 995–1028.
- Dominated Pesin theory: convex sum of hyperbolic measures. With Christian Bonatti and Katrin Gelfert. *Israel Journal of Mathematics*, 226 (2018), no. 1, 387–417.
- On the approximation of convex bodies by ellipses with respect to the symmetric difference metric. *Discrete & Computational Geometry*, 60 (2018), no. 4, 938–966.
- The basic ergodic theorems, yet again. *CUBO*, 20 (2018) no. 3, 85–95.

- Positivity of the top Lyapunov exponent for cocycles on semisimple Lie groups over hyperbolic bases. With M. Bessa, M. Cambrinha, C. Matheus, P. Varandas, and D. Xu. *Bulletin of the Brazilian Mathematical Society*, 49 (2018), no. 1, 73–87.
- A criterion for zero averages and full support of ergodic measures With C. Bonatti and L.J. Díaz. *Moscow Mathematical Journal*, 18 (2018), no. 1, pp. 15-61.
- The scaling mean and a law of large permanents. With Godofredo Iommi and Mario Ponce. *Advances in Mathematics* 292 (2016), 374-409.
- Robust criterion for the existence of nonhyperbolic measures. With C. Bonatti and L.J. Díaz. *Communications in Mathematical Physics* 344 (2016), no. 3, 751–795.
- Ergodic optimization of prevalent super-continuous functions. With Yiwei Zhang. *International Mathematics Research Notices*, 2016 (2016), no. 19, 5988–6017.
- Peano curves with smooth footprints. With Pedro H. Milet. *Monatshefte für Mathematik* 180 (2016), no. 4, 693–712.
- The entropy of Lyapunov-optimizing measures of some matrix cocycles. With Michal Rams. *Journal of Modern Dynamics* 10 (2016), 255–286.
- Continuity properties of the lower spectral radius. With Ian D. Morris. *Proceedings of the London Mathematical Society* 110 (2015), 477–509.
- Cocycles of isometries and denseness of domination. *Quarterly Journal of Mathematics* 66 (2015), no. 3, 773–798.
- A geometric path from zero Lyapunov exponents to rotation cocycles. With A. Navas. *Ergodic Theory and Dynamical Systems*, 35 (2015), no. 2, 374–402.
- Universal regular control for generic semilinear systems. With N. Gourmelon. *Mathematics of Control, Signals, and Systems*, 26 (2014), no. 4, 481–518.
- Almost reduction and perturbation of matrix cocycles. With Andrés Navas. *Annales de l'Institut Henri Poincaré – Analyse Non linéaire*, 31 (2014), no. 6, 1101–1107.
- Robust vanishing of all Lyapunov exponents for iterated function systems. With Christian Bonatti and Lorenzo J. Díaz. *Mathematische Zeitschrift*, 176 (2014), 469–503.
- Generic linear cocycles over a minimal base. *Studia Mathematica*, 218 (2013), no. 2, 167–188.
- Perturbation of the Lyapunov spectra of periodic orbits. With Christian Bonatti. *Proceedings of the London Mathematical Society*, 105 (2012), no. 1, 1–48.
- Nonuniform hyperbolicity, global dominated splittings and generic properties of volume-preserving diffeomorphisms. With A. Avila. *Transactions of the American Mathematical Society*, 364 (2012), no. 6, 2883–2907.
- Opening gaps in the spectrum of strictly ergodic Schrödinger operators. With A. Avila and D. Damanik. *Journal of the European Mathematical Society*, 14 (2012), no. 1, 61–106.
- Uniformly hyperbolic finite-valued  $SL(2, \mathbb{R})$  cocycles. With Artur Avila and Jean-Christophe Yoccoz. *Commentarii Mathematici Helvetici*, 85, no. 4 (2010), 813–884.
- $C^1$ -generic symplectic diffeomorphisms: partial hyperbolicity and zero center Lyapunov exponents. *Journal of the Institute of Mathematics of Jussieu*, 9, no. 1 (2010), 49–93.

- Nonuniform center bunching and the genericity of ergodicity among  $C^1$  partially hyperbolic symplectomorphisms. With Artur Avila and Amie Wilkinson. *Annales Scientifiques de l'École Normale Supérieure*, 42, n. 6 (2009), 931–979.
- Some characterizations of domination. With Nicolas Gourmelon. *Mathematische Zeitschrift*, 263, no. 1 (2009), 221–231.
- Cantor spectrum for Schrödinger Operators with potentials arising from generalized skew-shifts. With Artur Avila and David Damanik. *Duke Mathematical Journal*, 146, no. 2 (2009), 253–280.
- A uniform dichotomy for generic  $SL(2, \mathbb{R})$  cocycles over a minimal base. With A. Avila. *Bulletin de la Société Mathématique de France*, 135 (2007), 407–417.
- Generic expanding maps without absolutely continuous invariant  $\sigma$ -finite measure. With A. Avila. *Mathematical Research Letters*, 14 (2007), 721–730.
- A generic  $C^1$  map has no absolutely continuous invariant probability measure. With A. Avila. *Nonlinearity*, 19 (2006), 2717–2725.
- Dichotomies between uniform hyperbolicity and zero Lyapunov exponents for  $SL(2, \mathbb{R})$  cocycles. With B. Fayad. *Bulletin of the Brazilian Mathematical Society*, 37, no. 3 (2006), 307–349.
- A remark on conservative diffeomorphisms. With Bassam Fayad and Enrique Pujals. *Comptes Rendus Acad. Sci. Paris*, Ser. I 342 (2006), 763–766.
- The Lyapunov exponents of generic volume preserving and symplectic maps. With M. Viana. *Annals of Mathematics*, 161 (2005), No. 3, 1423–1485.
- $L^p$ -generic cocycles have one-point Lyapunov spectrum. With Alexander Arbieto. *Stochastics and Dynamics*, 3 (2003), 73–81.
- Inequalities for numerical invariants of sets of matrices. *Linear Algebra and its Applications*, 368 (2003), 71–81.
- Robust transitivity and topological mixing for  $C^1$  flows. With Flavio Abdenur and Artur Avila. *Proceedings of the American Mathematical Society*, 132 (2003), 699–705.
- Uniform (projective) hyperbolicity or no hyperbolicity: a dichotomy for generic conservative maps. With Marcelo Viana. *Annales de l'Institut Henri Poincaré – Analyse non linéaire*, 19 (2002), 113–123.
- Genericity of zero Lyapunov exponents. *Ergodic Theory and Dynamical Systems*, 22 (2002), 1667–1696.
- A formula with some applications to the theory of Lyapunov exponents. With Artur Avila. *Israel Journal of Mathematics*, 131 (2002), 125–137.

**Book chapters:**

- Ergodic optimization of Birkhoff averages and Lyapunov exponents. *Proceedings of the International Congress of Mathematicians 2018, Rio de Janeiro*, vol. 2, 1821–1842.
- Lyapunov exponents: How frequently are dynamical systems hyperbolic? With M. Viana. *Modern dynamical systems and applications*, 271–297, Brin, Hasselblatt, Pesin (eds.) Cambridge Univ. Press, 2004.
- Pisa Lectures on Lyapunov Exponents. With M. Viana. *Dynamical Systems – Part II: Topological, Geometrical, and Ergodic Properties of Dynamics*, 23–47. Scuola Normale Superiore, Pisa 2003.

**Preprints:**

The Halász-Székely barycenter. With Godofredo Iommi and Mario Ponce.  
arXiv:2103.05182

**Citations:** Cited 645 times by 312 authors, H number = 14, according to AMS MathSciNet (August 2021).

**Editorial Service**

Dec.2014 – Member of the editorial board of *Discrete and Continuous Dynamical Systems - Series A*.

**Teaching Experience****Advanced minicourses:**

*C<sup>1</sup> perturbation techniques in the neighborhood of periodic orbits*, Institute of Mathematics, Polish Academy of Sciences (Będlewo), 2013. 6 hour course, joint with Christian Bonatti.

*Quasi-periodic cocycles with Liouvillean frequencies*, Scuola Normale Superiore, Pisa, 2010. One week course, joint with Raphaël Krikorian.

*Lyapunov exponents*, ICTP, Trieste, 2008. Two week course, joint course with Artur Avila.

*Lyapunov exponents for generic volume preserving maps*, Morningside Center for Mathematics, Chinese Academy of Sciences, Beijing, 2007. One week course.

*Deterministic products of matrices*, Scuola Normale Superiore, Pisa, 2002. Joint course with Marcelo Viana.

**Graduate Teaching:**

Hyperbolic Dynamics: Penn State (2021), IMPA (2002).

Ergodic Theory: PUC-Chile (2017), PUC-Rio (2012), UFRGS (2006), IMPA (2003, 2002).

Complex Analysis: PUC-Chile (2016).

Measure Theory: PUC-Rio (2013, 2011), IMPA (2004).

Differential Topology: PUC-Rio (2011).

Analysis in  $\mathbb{R}^n$ : PUC-Rio (2010), IMPA (2003).

Dynamical Systems: PUC-Rio (2009, 2008)

General Topology: UFRGS (2006).

Algebraic Topology: UFRGS (2005).

**Undergraduate Teaching:**

Real Analysis: Penn State (2022), PUC-Chile (2016), PUC-Rio (2013, 2011, 2009), UFES Summer School (2006).

Precalculus: PUC-Chile (2021, 2019, 2018, 2017).

Differential Equations: PUC-Chile (2018), PUC-Rio (2011, 2010), UFRGS (2005).

Introduction to Dynamical Systems: PUC-Chile (2018, 2015).

Univariate Calculus: PUC-Chile (2020, 2017, 2016), PUC-Rio (2009), UFRGS (2007).

Lebesgue Integration: PUC-Chile (2015), UFRGS (2007).

Multivariate Calculus: PUC-Chile (2015, 2014), PUC-Rio (2012).

Differential Geometry: PUC-Chile (2014).

Discrete Mathematics: PUC-Rio (2013, 2010, 2009).

Linear Algebra: PUC-Chile (2019), PUC-Rio (2012, 2008), UFRGS (2006).

Introduction to Probability: PUC-Rio (2012, 2010).

Analytical Geometry: UFRGS (2007, 2006).

**Undergraduate Advising:** 2 students at PUC-Chile, 3 students at PUC-Rio, 1 student at UFRGS.

**School-level Teaching:**

Leader of the project *Mathematics for Highly Capable Students*, Anne Frank High School, Porto Alegre (2006).

## Graduate Students Advising

**Former students:**

Sebastian Pavez MSc, PUC-Chile. Dissertation: “Ergodic optimization and rotation sets.” (2020). Currently a PhD student at Penn State University.

Eduardo Oregón. MSc, PUC-Chile. Dissertation: “Negative curvature, matrix products, and ergodic theory” (2018). Currently a PhD student at University of California, Berkeley.

Renato Velozo. MSc, PUC-Chile. Dissertation: “Characterization of uniform hyperbolicity for fiber-bunched cocycles” (2018). Currently a PhD student at Cambridge University.

Paulo N. Orenstein. MSc, PUC-Rio. (Co-advised by Carlos Tomei). Dissertation: “Optimal transport and the Wasserstein metric” (2014). Currently a visiting professor at IMPA.

Zhou Cong. MSc, PUC-Rio. Dissertation: “Multiplicative ergodic theorem in non-positively curved spaces” (2013). Currently an assistant professor at Universidade Federal Fluminense.

Miguel K. Schnoor. PhD, PUC-Rio. Thesis: “The non-existence of absolutely continuous invariant probabilities is  $C^1$ -generic for flows” (2012). Currently an assistant professor at Universidade Federal Fluminense.

Pedro H. Milet. MSc, PUC-Rio. Dissertation: “Peano curves and line fields” (2011). Currently working at XP Investimentos.

## Academic Service

- 2021– Member of the Graduate Teaching Oversight Committee, Penn State.
- 2019 Advisor to the hiring committee, Mathematics Department, PUC-Chile.
- 2017 Member of the hiring committee, Mathematics Department, PUC-Chile.
- 2011–2013 Director of Graduate Studies, Mathematics Department, PUC-Rio.
- 2009 Member of the hiring committee, Applied Mathematics Department, Universidade Federal Fluminense (UFF).

## Event Organization (joint with other people)

- 2021 Dynamical Systems Session of the VI Latinamerican Congress of Mathematicians, Montevideo, Uruguay. (Online event).
- 2020 Special session “New Trends in Thermodynamic Formalism”, 31st Fall meeting of the Semi-annual Workshop in Dynamical Systems and Related Topics, Penn State. (Online event).
- 2018 Dynamics in Valparaíso, PUC-V. [ima.ucv.cl/congreso/dynamics-in-valparaiso](http://ima.ucv.cl/congreso/dynamics-in-valparaiso)
- 2013 Dynamical Systems Session of the 29th Brazilian Mathematical Colloquium (IMPA).
- 2010–2012 EDAI (monthly Dynamical Systems joint seminar of PUC-Rio, UFRJ and UFF universities). [www.mat.puc-rio.br/edai](http://www.mat.puc-rio.br/edai)
- 2010 VIII Oktobermat, PUC-Rio. [www.mat.puc-rio.br/oktober2010](http://www.mat.puc-rio.br/oktober2010)



## **Supplementary Information**

Born June 12, 1975 in Porto Alegre, Brazil.

Married, two children (born 2017 and 2019).

January 21, 2022